Module 3: Prompting and Supporting Argumentation – Focus on Implementation: Norms and Routines

June 29, 2016

Module Goals:

- Develop a deeper understanding of argumentation and its potential in the math classroom.
- Examine norms and routines that can support mathematical argumentation in the classroom.
- Develop an understanding of a pedagogy of inquiry to support mathematical argumentation in the classroom.

The formulation of a problem is often more important than its solution, which may be merely a matter of mathematical or experimental skill.

- Albert Einstein

Animal Populations

Algebra –SSE

Original problem from illustrativemathematics.org

Suppose P and Q give the sizes of two different animal populations, where Q > P. In (a) – (d), say which of the expressions is larger. Briefly explain your reasoning in terms of the two populations.

(a) $P + Q$ and $2P$

(b) $(Q - P)/2$ and $Q - P/2$

(c) $\frac{P}{P+Q}$ and $\frac{P+Q}{2}$

(d) $P + 50t$ and $Q + 50t$

Class Background for Video Clip:
Establishing Norms for Argumentation

The students in the clip are six 6th graders to whom I provided support and intervention several days a week in their math class. On Fridays, I pulled them altogether to another room to focus on Mathematical Practice 3, along with Sharon Heyman, who was at UConn and working on the Bridges Project. These students were struggling in math at the time. My focus of instruction around argumentation and critiquing the reasoning of others began between late January and early February. The clip is from a lesson near the end of March.

My initial goal was to engage students in forming a mathematical argument through discourse and writing. As with many students who struggle with math, these students hesitated to engage in mathematical discourse. So, I decided we needed to create a community agreement to set some guidelines. Because the students had very little experience in mathematical discourse my focus became more about strengthening mathematical discussion by modeling what this process should look like in a typical 6th grade class. Sharon and I felt that getting them to engage in discourse and critiquing the reasoning of the others was going to be crucial to have them start the process of writing an argument.

After having the students create their community agreement, we showed them a video of other 6th grade students and asked them to notice what the students were doing and what the teacher was doing. They noted that the students did most of the talking and the teacher mainly asked questions. The second step was to model how discourse between two people who may have solved a problem with different strategies might look. I had the students use a discourse checklist to “score” the math discussion Sharon and I had after we solved a mathematical argument problem. We continued to use this checklist during our Friday sessions for student-to-student discourse. The third step was for me to be mindful about being explicit with my mathematical vocabulary and asking students many clarifying questions while they worked together in pairs or shared their thinking to the entire class. I also reminded the students that I was not looking for them to explain step-by-step how they solved a problem, but *why* they solved it the way they did. At this point, my focus wasn’t necessarily whether or not they got an answer correct, it was to provide an arena for them to feel safe enough to discuss their math thinking. Overtime we looked at ways they could help each other arrive at a correct answer.

It is also important to note that each session began with a type of “energizer” task that encouraged students to problem solve, reason and engage in mathematical discourse. This might have been a number talk, “can this be true” problem, or critiquing an incorrect answer from previous sessions.

When we first started these Friday sessions, several things were apparent. The students had misconceptions about how to solve some of the problems in multiple ways, limited ability to justify their answers, didn’t know how to explain their answer (the work) verbally. If they got a problem wrong, often they didn’t look for how their answer was incorrect and then make adjustments. They hardly ever critiqued the reasoning of others. By the end of the school year, however, I saw lots of growth in almost all six students. One student, Chamari, who is highlighted in the video showed the most growth, and in so many ways. Both Sharon and I saw his confidence level grow, he was able to clearly state how he solved a problem, he frequently shared his thoughts regarding the others students solutions and was starting to show progression in his math class.

Video Viewing
Establishing Norms for Argumentation
Guiding Questions

Please feel free to jot notes here or mark on the transcript.

1. What moves do you see the teacher making to help promote a culture of thinking?

2. What do you see students saying or doing that relates a culture of thinking?

3. What norms may have been previously established in this class to support this interaction?

Ms. McKnight's Classroom Clip
Exploring Classroom Norms

1	Ms. M:	So do you want to go first Chamari?
2	Chamari:	I can't draw well.
3	Ms. M:	You did a great job. So, Marion and Ari, now it's time to listen. You can draw on the board, I'll give you time to do that before you leave. Ari?
4	Ari:	Yeah?
5	Ms. M:	Be respectful, please. I said I'll give you time. So, can you guys find a place to sit? And Chamari is gonna talk about his idea. Okay. So, Chamari can you make sure all of us can hear. So, just tweak your body a little, and look at us.
6	Chamari:	[turns to face whiteboard]
7	Ms. M:	Look at *us*.
8	Chamari:	[Turns around to the other side and looks out towards the class] Okay. First, I drew the rect– I drew the boxes. Then, I…
9	Ms. M:	So – I'm gonna – I'm only stopping you… And this is why I'm stopping him, because remember, we're going to try to stay away from telling step-by-step what we did. We want to talk about *why* we did that. So, why did you decided to do what you did? Instead of telling us step-by-step, because we didn't see what you did. How does it compare six-tenths – what you did – compare six-tenths and four-sixths?
10	Chamari:	I chose this way because I thought it was easier. Because if I use the one bar then it would be easier to draw the denominators.
11	Ms. M:	Okay. Can you hear him? Please stop drawing. Go ahead.
12	Chamari:	Because… Like, if I drew – if I drew – use the one bar, then it would b easier to um… draw the denominators.
13	Ms. M:	What about those denominators? What can you tell me about those denominators?
14	Chamari:	Ah… They're both different, but they're both the same um… size.
15	Ms. M:	So they're both different. Everyone agree with the denominators are different?
16	Student 1:	I can't see.
17	Ms. M:	Well, the denominators are the same as what you have. Six-tenths and four-sixths. Ten and six. Right?
18	Student 1:	I said, but yeah.
19	Ms. M:	Okay. Are they the same size? The denominator tenths and the denominator sixths? Are those the same size?
20	Students:	No
21	Ms. M:	[Walks over to pick up fraction bar manipulatives] So if I look at some

fraction bars – a sixth and a tenth – are those the same size?

22 Students: No.

23 Ms. M: They're not. So would you still stick with that, Chamari? That they're same size? So, they're different, right? We know that, for sure.
[…]

24 Ms. M: Alright. So, what was your conclusion? I see that you wrote down six-tenths. What is your conclusion? Which one's greater? Which one's less? Or are they equal?

23 Chamari: Ah, four-sixths is greater because four is closer – closer to the denominator… And six is further.

25 Ms. M: Okay. So let me just make sure I understand. You're saying that four-sixths is the bigger fraction – it's greater. Does anyone know how I would write four-sixths is greater?

26 Chamari: Yeah.

27 Ms. M: So. Does anyone know how I would write four-sixths is greater?

28 Ari: Yes.

29 Ms, M: Can you come up and show me? And so, Trinity and Michael and Jason, can you look over her?

30 Ari: [Walks to the whiteboard and fills in the symbol so the comparison looks like: $\frac{6}{10} < \frac{4}{6}$]

31 Students: No. Yes.

32 Ms. M: So, who can tell me which one says that four-sixths is greater? The top one or the bottom one?

33 Student: Bottom.

34 Ms. M: Why?

35 Student: Because, the – where the…

36 Ms. M: So, this is a symbol.

37 Student: [overlapping explanations]

38 Ms. M: Yeah. And we can make it as simple as that, right? We can say… And that's just how I remember it. Like, oh yeah, this is like the bigger size … And like kind of chomping down on the bigger. I think you're thinking – I know what you're thinking here. You just have know how to write the symbol. Okay? So, who else wants to explain or maybe agree or disagree? Chamari is saying that six-tenths is less than 4/6.
[Class discussion continues]

This document describes general norms and expectations about mathematics that can help to promote a culture of thinking in math class.

Positive Norms to Encourage in Math Class

By Jo Boaler

http://www.youcubed.org/wp-content/uploads/Positive-Classroom-Norms2.pdf

1. Everyone Can Learn Math to the Highest Levels.
Encourage students to believe in themselves. There is no such thing as a "math" person. Everyone can reach the highest levels they want to, with hard work.

2. Mistakes are valuable
Mistakes grow your brain! It is good to struggle and make mistakes.

3. Questions are Really Important
Always ask questions, always answer questions. Ask yourself: why does that make sense?

4. Math is about Creativity and Making Sense.
Math is a very creative subject that is, at its core, about visualizing patterns and creating solution paths that others can see, discuss and critique.

5. Math is about Connections and Communicating
Math is a connected subject, and a form of communication. Represent math in different forms eg words, a picture, a graph, an equation, and link them. Color code!

6. Depth is much more important than speed.
Top mathematicians, such as Laurent Schwartz, think slowly and deeply.

7. Math Class is about Learning not Performing
Math is a growth subject, it takes time to learn and it is all about effort.

Norms & Expectations for Argumentation in Partners
Progress Across the Year in a 3rd Grade Class

Early in the School Year	• Facing each other, hands and legs quiet. • Looking at each other. • One person shares their claim, evidence and warrant and the other person listens. • The other person can say or write exactly what the other person has just told them. • Repeat with the other person
	• One person shares their claim, evidence and warrant and the other person listens. • The other person can say or write in their own words and ask questions what the other person has just told them. • Repeat with the other person.
	• One person shares their idea and the other person listens. • The other person agrees or disagrees with the other person's claim, evidence and/or warrant and explains why. (Concentrate on agreeing or disagreeing with the argument, NOT the person.) • Repeat with the other person.
Later in the School Year	• Each person shares their claim, evidence, and warrant. • They ask each other clarifying questions. • They ask each other to explain their evidence in a different way. • They agree or disagree with each other and state why using their evidence. • They build upon what they are learning from their partner's argument and improve or change their own argument.

Questions a teacher can ask to help promote argumentation

Can you explain?	What does that look like?
What happened before?	What happened after?
What would happen if you used this number?	Can you tell me why…?
What would change if...	Tell me more.
Show me where...	How does this fit?
What could you add to strengthen this part?	Why did you use that equation?
How would that work?	So you are saying…?

Modified based on resources created by Sarah Brown

ATOMIC 2015 Presentation: *What is Mathematical Argumentation?*

Some Additional Resources for Norms and Expectations in the Mathematics Classroom

This handout contains excerpts from three resources focused on supporting discussion and reasoning in mathematics classrooms.

1) Excerpt from *Classroom Norms for Productive Discourse & Discussion*
(O'Connor & Ruegg, 2012)

Classroom discourse and discussion are crucial sites for student development: they can help students develop the ability to be critical thinkers, and to feel more confidence in their own abilities to construct arguments and to understand the arguments of others.

Nevertheless, classroom talk and discussion can be challenging. Teachers who tend to have success with classroom discourse and discussion usually have worked out dependable ways to make sure that the discourse is *respectful, equitable*, and *focused on reasoning*. They make sure that they set up norms of productive discussion within the first week or so of school. This usually includes more than posting norms on the wall. It usually includes a discussion about what is involved, to get student understanding and buy-in.

What is included in setting up norms for discussion?
In setting up norms, you will be letting your students know about how talk and discussion will play a role in their learning during the coming year. You are setting up expectations, and so your students need to understand those expectations. Depending on the ages and stages of your students, it is most helpful if you can enlist their participation in co-constructing those expectations. Consider having a discussion about these three different kinds of discussion norms and goals:

1) Talk that is focused on *reasoning*
2) Talk that is *respectful*
3) Talk that is *equitable*

Here is an example of norms for discussion that an eighth grade teacher co-created with her students:

- We come prepared for discussions with notes, examples, stories, texts.
- We are active participants, responsible for our own learning. This means we speak, request clarification, show agreement or confusion, verify, ask others to repeat.
- We strive to have authentic discussions that are academically rich. This means we stay on topic and ask what we really want to know.
- We push ourselves and each other to think beyond the obvious, popular, or easy answers. This means we request proof or reasoning, point out misinformation, disagree with parts, draw others out, and are open to changing our minds.

2) Excerpt from “Revoicing: The good, the bad, the questions” by Jean Krusi (High School Algebra Teacher) in the collection *Promoting Purposeful Discourse* (Herbel-Eisenmann & Cirillo, 2009, p. 121)

As I thought about how I wanted to change the discourse patterns in my classroom, I decided I wanted to involve my students in the process. I started by asking them what made a good classroom discussion. The students seemed to be importing ideas about class discussion from their other classes, possibly indicating that they did not think of mathematics classes as places for rich discussion. We used the ideas of what makes a good discussion, more generally, to produce the following set of discussion norms for our classroom:

- Everyone is listening. Everyone is involved. Everyone puts out ideas. No one is left out.
- No one is talking while someone else is. Take turns.
- Questions are asked. Make your point clearly and quickly. Have facts to back up your point. It is safe to be wrong.
- No rude comments or put-downs. All ideas and opinions are respected. Different points of view are valued. "Out of the box" thinking can be helpful.
- Everyone is understanding-if not at the beginning, then by the end.

Most of these norms are social norms. Some, such as "have facts to back up your point" and "everyone is understanding," do have the potential to suggest mathematical norms. During our discussions, I found that I needed to bring up mathematical norms, such as what counts as evidence in mathematics or the idea of generalizing statements. In the future, I would like to try having a discussion about elements particular to mathematical discourse as a class, with the idea of generating mathematical norms as well as social norms.

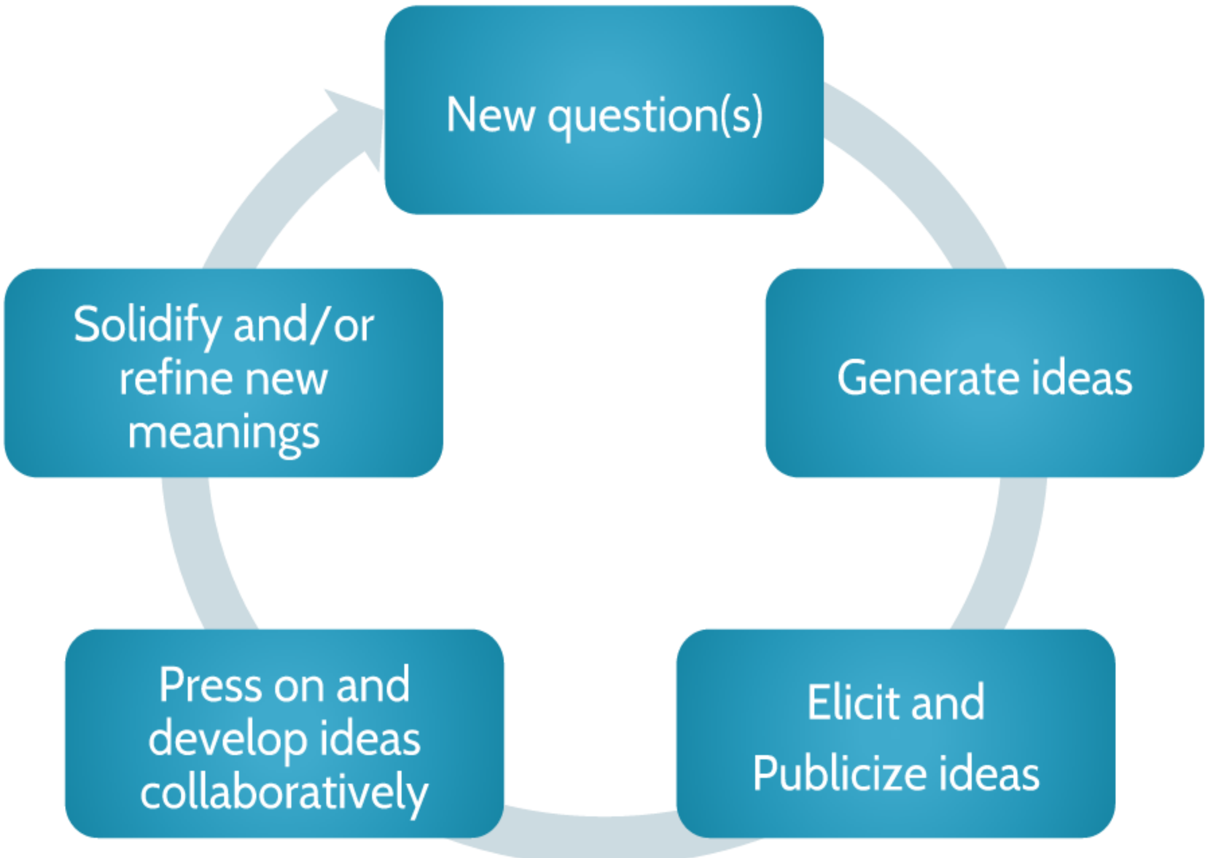
3) Excerpt from the book *Making Number Talks Matter* about how to make a safe classroom environment (Humphreys & Parker, 2015, p. 169)

Our bottom line is that we want the learning environment to be safe for all students. Ruth shares, “I tell my students on the first day of class that I won’t put them on the spot but that I will give them lots of opportunities to share their thinking when they choose to. I try hard not to violate this trust. I want the learning environment to be safe for all students. I do talk with kids about how important it is for them to talk about and explain their thinking. With quieter students, I sometimes ask them, one-on-one, to share with me how they thought about a problem. Once they have had a chance to rehearse their thinking with me, I ask them to think about whether they might be willing to share their ideas. Once students have had their own way of thinking recognized and valued, they may become more confident in sharing their ideas.”

PEDAGOGY

What routines or instructional strategies may be used in the classroom to support a culture of thinking and mathematical argumentation?

A Pedagogical Model to Support a Culture of Thinking



Chain of Flowers
Pattern Task

Consider the following pattern:

Figure 1 Figure 2 Figure 3

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a. Draw Figure 5. How many tiles does it have?

b. How many tiles will the 25th figure have? How do you know?

c. How many tiles are in the *n*th figure? How do you know?

Debriefing the Talk Frame Routine
Flower Pattern Task

1. How does the Talk Fame allow students to generate ideas?

2. How can the teacher facilitate this process?

3. How does the teacher facilitate this process in order to promote thinking?

4. As the class develops ideas, what is the ultimate goal of the teacher?

5. How is that goal achieved based on student responses?

6. What does the teacher do to help students make connections between ideas and solidify their learning?

Talk Frame Teacher Template

Think

Talk Idea

Talk Idea

Talk Idea

We Understand

____

____

____

____

____

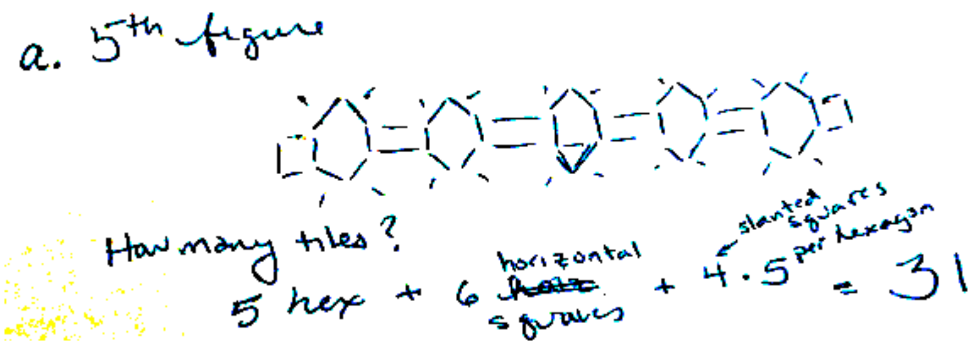
Talk Frame Teacher Template

Think

- How many tiles in the 5^{th} figure?
- How many tiles in the 25^{th} figure?
- How many tiles in the n^{th} figure?

Talk Idea

Pictorial Representation:



Talk Idea

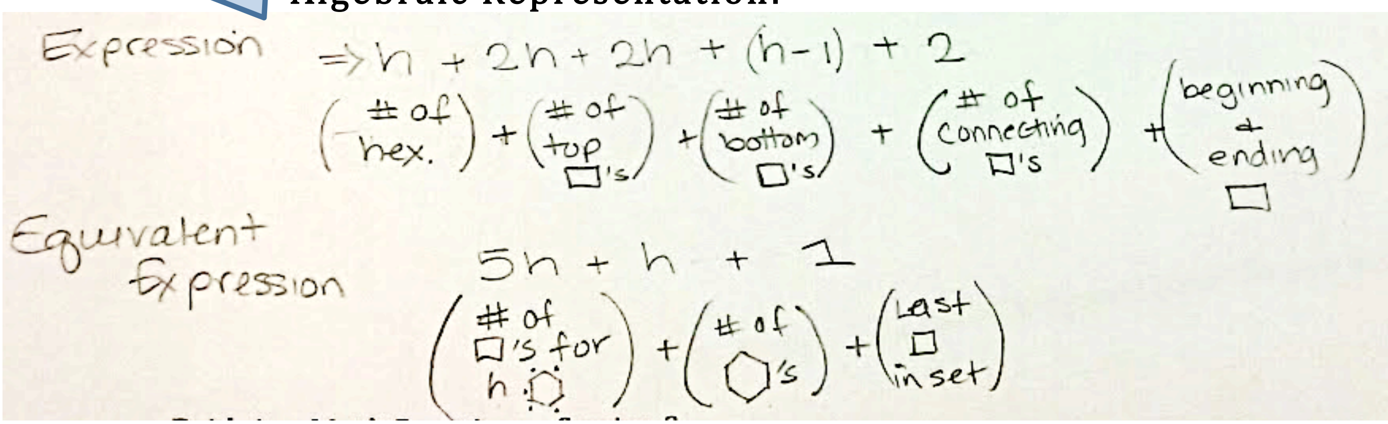
Numerical/Tabular Representation:

Figure	tiles
1	7
2	13
3	19
4	25
5	31

I got this by following the pattern. For figure 1, 2, 3, you see that it adds 6 each time. Following that pattern, 4th is 25 and 5th is 31.

Talk Idea

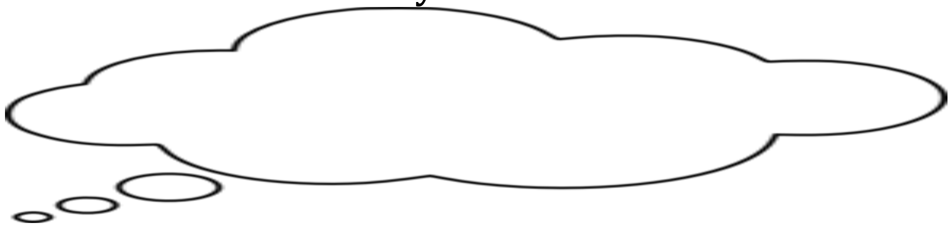
Algebraic Representation:



We Understand

- ***h*** represents the number of hexagons in the figure
- There are equivalent expressions to represent the solution
- The non-algebraic representations can be translated in to the algebraic.

Please record your responses to the following questions during lunch and place them in the folder at your table.



1. *Which norms/routines do you see yourself using at the beginning of the school year? Why?*

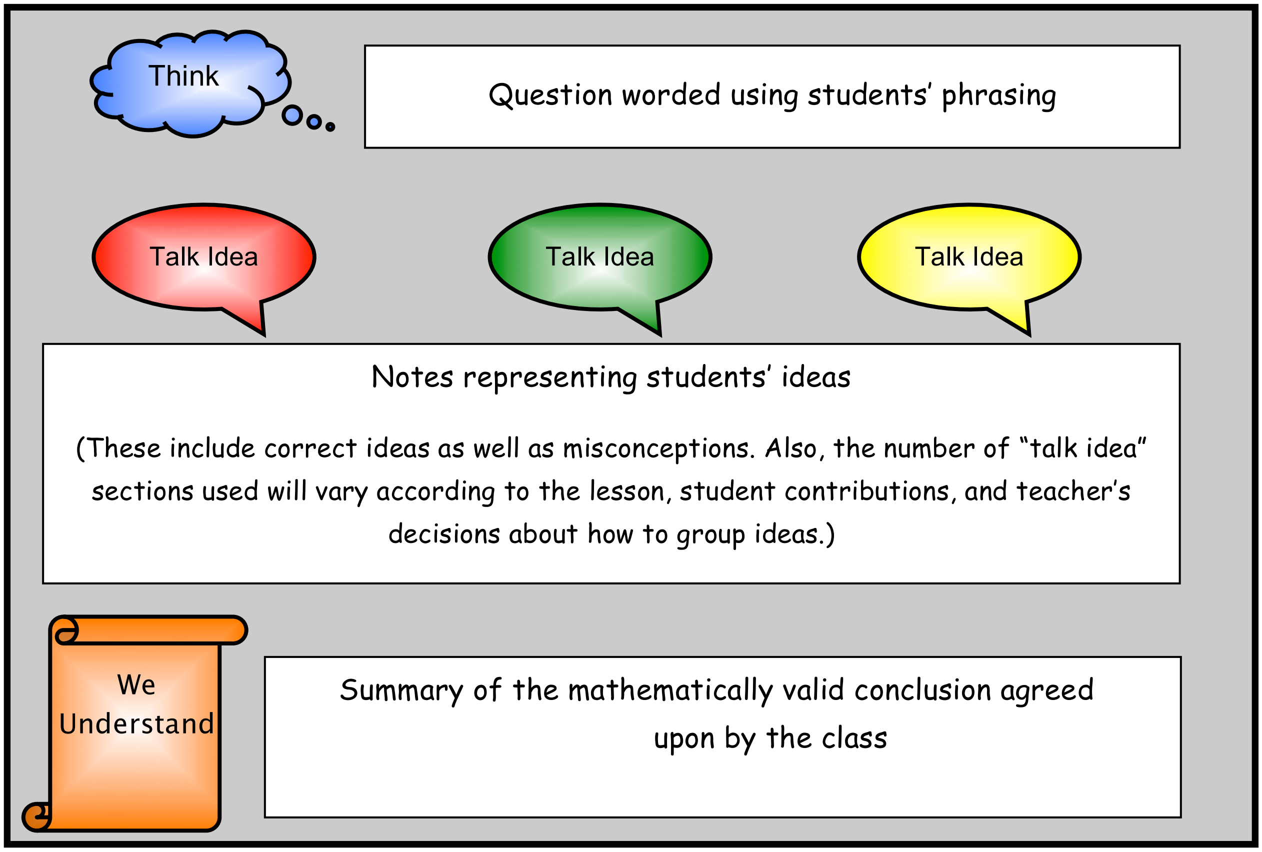
2. *All students can and should have opportunities to participate in argumentation. How might that work differ from class to class, or across groups of students with different needs?*

3. *What have you tried already that has worked well in your classroom?*

Please record your responses to the following questions during lunch and place them in the folder at your table.

Overview of the *Talk Frame* Pedagogical Routine

This *Talk Frame,* and other similar pedagogical "tools," is a routine that can be used to help organize mathematical discussions, keep track of student contributions, and synthesize the mathematical ideas. Note that the teacher can also introduce an idea using another idea card, not shown here.



From *Exploring Shape Games: Geometry with Imi and Zani*, by M. K. Gavin, T. M. Casa, S. H. Chapin, and L. J. Sheffield. Copyright © 2012, by Kendall Hunt Publishing Company.

For teachers, the Talk Frame can:

- Help focus the discussion on a significant mathematical topic;
- Encourage you to concentrate on and make sense of students' ideas;
- Allow you to formatively assess the class's and individual student's depth of understanding; and
- Assist in facilitating the process for the class to come to valid mathematical conclusions.

For students, the talk frame can:

- Allow them to see their thinking develop over time;
- Encourage them to rely on their own reasoning;
- Convey that their ideas are important, yet they can change;
- Let them realize that there can be multiple perspectives when solving problems;
- Have them see how different representations, such as drawings, can symbolize ideas;
- Showcase the appropriate use of math vocabulary; and
- Reinforce the eventual need for mathematical agreement.

Talk frame. It can be challenging for both teachers and students alike to keep track of all that has been said during a discussion. Developed as part of the *Project M^2* units, a *talk frame* helps to keep track of student contributions and revisions to their ideas. The talk frame is a system that helps teachers organize and maintain a more permanent record of the conversation on the board. There are three sections to the talk frame using distinct icons:

1. Establish the topic: Teachers first need to identify the topic of discussion. It should be based on an important mathematical idea of the lesson and one for which students probably will be able to offer different ideas (either because there are multiple solution paths, various correct answers, or current misconceptions and misunderstandings among students).

2. Have students offer, consider and clarify ideas:
Teachers then need to gather different student perspectives. It is important for teachers not to judge their validity at this point, either verbally or through gestures, since students should be encouraged to make sense of the mathematical concepts themselves. Do not erase student ideas as they revise them. A record of the entire discussion should be produced. If necessary, teachers can introduce different perspectives they want students to contemplate through Zani (a character used in the *Project M^2* units). It is important that Zani introduces *both* correct and incorrect ideas; otherwise students may assume that anything presented by them is incorrect. Through this entire process, students will clarify their ideas both for themselves and others.

3. Settle on a mathematical understanding: Teachers eventually will guide students to reach a mathematical understanding. Often, students will notice the errors in their thinking given time to debate and justify their ideas. In this section, the new understandings are recorded. Teachers may elicit multiple "we understands" from students, or prompt the class to articulate one key understanding based on the discussion.

Talk Frame Model - planning template

Think

Question worded using students' phrasing:

Without using the traditional algorithm, can you make sense of 1 ÷ ⅔ ?

Talk Idea

Talk Idea

Talk Idea

Anticipated ideas

(Diagrams & pictures are likely also)

1/3 goes into 1 three times & 2 times 3 equals 6. So the answer is 6

Dividing by 2/3 is the same as multiplying by its reciprocal (3/2) That's the same as multiplying by 3 and then dividing by 2. 1 x 3=3 and 3 divided by 2= 1 ½

How many 2/3s fit into 1? On a number line, one 2/3 fits & then another 1/3 fits. 1/3 is half of 2/3, so one & a half 2/3s fit into 1

What's a potential goal(s) for the discussion? What should students come to better "understand"?

We Understand

- We can explain fractions division problems in ways that make sense.
- We can figure out if an answer (or method) to a fractions division problem is reasonable.
- We agree that 2/3 can fit into 1 one time with 1/3 left over and 1/3 is ½ of 2/3 so 1 ÷ ⅔ = 1 ½

Your Turn: Talk Frame Mini-Lesson

1) With your team
    - Select a problem task (see separate sheet for choices)
2) Individually
    - Do the problem
    - Write down ideas that you would anticipate coming up
    - Put stars by ideas to bring to the group for discussion
3) With your team
    - Talk about the problem
        - What's worth discussing related to this problem?
        - What would make a good "talk frame" question? (You may reword the problem and/or select a part of the problem)
        - What would be an important goal for the talk frame discussion?
    - Use the Talk frame template to help you prepare a mini-lesson (with your team)
        - Consider student friendly language for the focusing question/problem
        - Anticipate ideas that you think may come up
        - Record a potential goal (or goals) of the talk frame discussion
    - Talk with your team about how you will teach this mini-lesson (who will do what, etc.)
4) With another (assigned team)
    - Teach the mini-lesson
    - Let another team teach you their mini-lesson

Talk Frame Model - planning template

Think

Question worded using students' phrasing:

Talk Idea

Talk Idea

Talk Idea

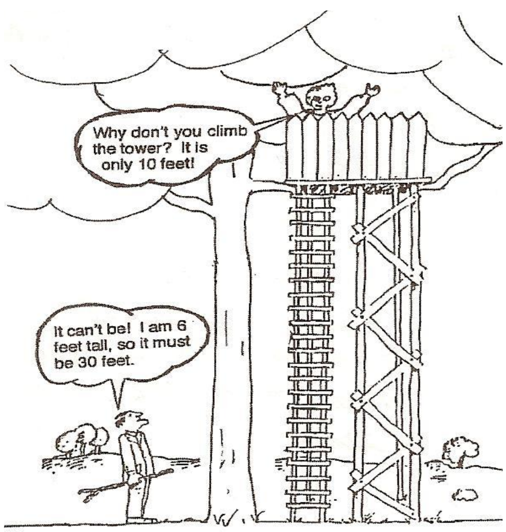
Anticipated ideas

What's a potential goal(s) for the discussion? What should students come to better "understand"?

We Understand

GROUP A (Elementary):

1) With your team, select one of these problems
2) Individually work the problem
3) With your team, talk about the problem
    - What's worth discussing in a mini-lesson related to this problem?
    - What would make a good "talk frame" question related to this problem? (You may reword the problem and/or select some part of the problem for the talk frame mini-lesson)
4) Use the Talk frame template to help you prepare a mini-lesson
5) Talk with your team about how you will teach this mini-lesson (who will do what, etc.)
6) Teach the mini-lesson to your assigned team **(Group B)**



Tree House Problem

These people disagree on the height of the tree house. How high do you think it is? Explain your reasoning.

(The person in the tree house says, "Why don't you climb the tower. It is only 10 feet tall." The person on the ground says, "It can't be! I am 6 feet tall, so it must be 30 feet.")

(from Lamon's *Teaching fractions and ratios for understanding* book)

Lemonade

Liam and Doug both need 18oz of lemon juice to make lemonade. Each Lemon makes about 1 ½ ounces of lemon juice. Liam says he needs 10 lemons to make enough lemon juice and Doug says he needs 12 lemons.

Do you agree with either student? Why or why not? Explain your reasoning.

(from UConn Repository-grade 4)

GROUP B (Elementary):

1. With your team, select one of these problems
2. Individually work the problem
3. With your team, talk about the problem
    - What's worth discussing in a mini-lesson related to this problem?
    - What would make a good "talk frame" question related to this problem? (You may reword the problem and/or select some part of the problem for the talk frame mini-lesson)
4. Use the Talk frame template to help you prepare a mini-lesson
5. Talk with your team about how you will teach this mini-lesson (who will do what, etc.)
6. Teach the mini-lesson to your assigned team **(Group A)**

School Days Problem

Alec and Felix are brothers who go to different schools. The school day is just as long at Felix's school as at Alec's school. At Felix's school, there are 6 class periods of the same length each day. Alec's day is broken into 3 class periods of equal length.

One day, it snowed a lot so both of their schools started late. Felix only had four classes and Alec only had two. Alec claims his school day was shorter than Felix' was because he had only two classes on that day. Is he right? Explain how you know.

(from illustrativemathematics.org – grade 3)

Student Height Problem

Mr. Liu asked the students in his fourth-grade class to measure their heights. Here are some of the heights they recorded.

Student	Height
Sarah	50 inches
Jake	$4\frac{1}{4}$ feet
Andy	$1\frac{1}{2}$ yards
Emily	4 feet and 4 inches

List the students from tallest to shortest. Explain how you figured the order.

(adapted from illustrativemathematics.org – grade 4)

GROUP C (Elementary/Middle):

1. With your team, select one of these problems
2. Individually work the problem
3. With your team, talk about the problem
   - What's worth discussing in a mini-lesson related to this problem?
   - What would make a good "talk frame" question related to this problem? (You may reword the problem and/or select some part of the problem for the talk frame mini-lesson)
4. Use the Talk frame template to help you prepare a mini-lesson
5. Talk with your team about how you will teach this mini-lesson (who will do what, etc.)
6. Teach the mini-lesson to your assigned team **(Group D)**

Piggy Bank Problem

Alicia opened her piggy bank and counted the coins inside. Here's what she found:

- 22 pennies
- 5 nickels
- 5 dimes
- 8 quarters

a) What fraction of the coins are dimes? Explain how you know.

b) What fraction of the total value of the coins in the piggy bank is made up of dimes? Explain how you know.

c) Alicia realizes she looked too quickly. One of the nickels is really a quarter. Would this change your answer to either part a or part b? Explain why or why not.

(adapted from illustrativemathematics.org – grade 4)

Ellen's Rule Problem

Ellen says:
When you multiply by a number, you will always get a bigger answer.
Look, I can show you.

- **Start with 9. Multiply by 5.**
- **9x5=45**
- **45 > 5**

It even works for fractions

- **Start with ½. Multiply by 4.**
- **½ x 4 = 2**
- **2 > ½**

Does Ellen's rule always work? Explain your reasoning.

(adapted from Illustrativemathematics.org – grade 5)

GROUP D (Middle/Secondary):

1. With your team, select one of these problems
2. Individually work the problem
3. With your team, talk about the problem
    - What's worth discussing in a mini-lesson related to this problem?
    - What would make a good "talk frame" question related to this problem? (You may reword the problem and/or select some part of the problem for the talk frame mini-lesson)
4. Use the Talk frame template to help you prepare a mini-lesson
5. Talk with your team about how you will teach this mini-lesson (who will do what, etc.)
6. Teach the mini-lesson to your assigned team **(Group C)**

Chichén Itzá Problem

Chichén Itzá was a Mayan city in what is now Mexico. The picture below shows El Castillo, also known as the pyramid of Kukulcán, which is a pyramid located in the ruins.

The temple at the top of the pyramid is approximately 24 meters above the ground, and there are 91 steps leading up to the temple. How high above the ground would you be if you were standing on the 50th step?

(adapted from Illustrativemathematics.org – grade 6)

Job Earnings Problem

Kell works at an after-school program at an elementary school. The table below shows how much money he earned every day last week.

	Monday	Wednesday	Friday
Time worked	1.5 hours	2.5 hours	4 hours
Money earned	$12.60	$21.00	$33.60

Mariko has a job mowing lawns that pays $7 per hour.

Who would make more money for working 10 hours? Explain how you know.

Who makes more money per hour? How much more per hour?

(adapted from Illustrativemathematics.org – grade 8)

GROUP E (High School):

1. With your team, select one of these problems
2. Individually work the problem
3. With your team, talk about the problem
   - What's worth discussing in a mini-lesson related to this problem?
   - What would make a good "talk frame" question related to this problem? (You may reword the problem and/or select some part of the problem for the talk frame mini-lesson)
4. Use the Talk frame template to help you prepare a mini-lesson
5. Talk with your team about how you will teach this mini-lesson (who will do what, etc.)
6. Teach the mini-lesson to your assigned team **(Group F)**

Which is more square?

Which is *more square*, a rectangle that measures 35" x 39" or a rectangle that measures 22" by 25"? Explain how you know.

(from Lamon's *Teaching fractions and ratios for understanding* book)

The Mystery Bags Game

A king loves to play the Mystery Bags game. First, the jester takes one or more empty bags and fills them each with the same amount of gold. Next, he takes out his pan balance and places some combination of bags of gold and one-ounce weights on each of the pans. The king then tries to figure out how many ounces of gold are in one mystery bag.
You have been asked to help the king play the game. Without first translating to algebraic notation, figure out how much gold is in each bag. Explain your reasoning.

a) There are 8 bags of gold and 10 ounces on one side and 90 ounces on the other side.
b) There are 11 bags of gold and 65 ounces on one side and 4 bags of gold and 100 ounces on the other side.
c) There are 15 bags of gold and 7 ounces on both sides. (At first the king thought this would be easy, but then found it to be incredibly hard)
d) There are 4 bags of gold and 8 ounces on one side and 6 bags of gold and 12 ounces on the other side.

Adapted from *Interactive Math Program* 1 (IMP1), 1994

GROUP F (High School):

1. With your team, select one of these problems
2. Individually work the problem
3. With your team, talk about the problem
    - What's worth discussing in a mini-lesson related to this problem?
    - What would make a good "talk frame" question related to this problem? (You may reword the problem and/or select some part of the problem for the talk frame mini-lesson)
4. Use the Talk frame template to help you prepare a mini-lesson
5. Talk with your team about how you will teach this mini-lesson (who will do what, etc.)
6. Teach the mini-lesson to your assigned team **(Group E)**

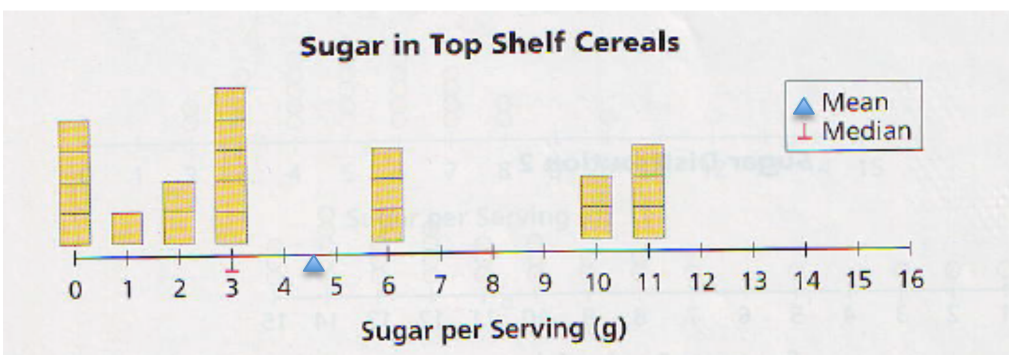
Traffic Jam Problem

Last Sunday an accident caused a traffic jam 12 miles long on a straight stretch of a two-lane freeway. How many vehicles do you think were in the traffic jam? Explain your reasoning.

(Adapted from Illustrative Mathematics – High School – Number & Quantity)

Sugar on the Shelf

Shown in the graph is the number of grams of sugar per serving for 20 breakfast cereals that are commonly found on the top shelf in the grocery store. The mean and median for the amount of sugar has also been marked.



a. Suppose you remove the three cereals with 6 grams of sugar per serving and add three new cereals, each with 9 grams of sugar per serving. What happens to the mean and the median? Why do you think this happens?

b. Use the new distribution from part (a). Suppose you remove a cereal with 3 grams of sugar and add a cereal with 8 grams of sugar. How do the mean and the median change?

c. Suppose you remove another cereal with 3 grams of sugar and add another cereal with 8 grams of sugar. How do the mean and the median change?

d. Suppose you remove a third cereal with 3 grams and add a third cereal with 8 grams of sugar. How do the mean and the median change? Explain how you know.

(Adapted from CMP)