Which Figure Is It? (Geometry)
ANNOTATED
STUDENT WORK SAMPLE
ARGUMENTATION RESOURCE
PACKET

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much "weight" people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional discussions and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

Student 3

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

Parallelogram, Rectangle, Rhombus, Square, Trapezoid

Use slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$\overline{AB}$: $m = \dfrac{5-2}{4+5} = \dfrac{3}{9}$ ⟨$\frac{1}{3}$⟩

$d = \sqrt{(4+5)^2 + (5-2)^2}$

$d = \sqrt{81 + 9}$

$d = \sqrt{90}$

⟨$d = 3\sqrt{10}$⟩

$\overline{DC}$: $m = \dfrac{-1+4}{6+3} = \dfrac{3}{9}$ ⟨$\frac{1}{3}$⟩

$d = \sqrt{(9)^2 + (3)^2}$

$d = \sqrt{90}$

⟨$d = 3\sqrt{10}$⟩

$\overline{BC}$: $m = \dfrac{5+1}{4-6} = -\dfrac{6}{2}$ ⟨-3⟩

$d = \sqrt{(-2)^2 + (6)^2}$

⟨$d = 2\sqrt{10}$⟩

$\overline{AD}$: $m = \dfrac{2+4}{-5+3} = -\dfrac{6}{2}$ ⟨-3⟩

$d = \sqrt{(-2)^2 + (6)^2}$

$d = \sqrt{4 + 36}$

$d = \sqrt{40}$

⟨$d = 2\sqrt{10}$⟩
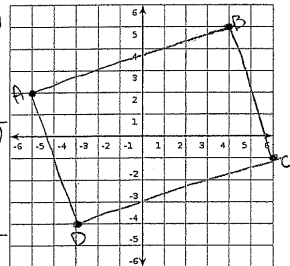
This quadrilateral is a rectangle

By definition, a rectangle is a quadrilateral in which there are 2 sets of sides that are the same length and parallel and 4 right angles. There are 2 sets of sides that are parallel + the same length. $\overline{AB}$ is parallel to $\overline{CB}$ with the slope $\frac{1}{3}$, and both are the same length ($3\sqrt{10}$). $\overline{AD}$ is parallel to $\overline{BC}$ with the slope -3, and both of these lines are also the same length ($2\sqrt{10}$). I also know that this figure has all right angles because $\overline{AD}$ and $\overline{BC}$, with slopes of -3, are perpendicular to lines $\overline{AB}$ and $\overline{CB}$, with slopes of $\frac{1}{3}$. When lines are perpendicular, they create right angles. Based on all of this information, this quadrilateral can be proven to be a rectangle.

Commentary

This student's argument was categorized as **HIGH quality.**

This student makes the claim of Rectangle. The student's argument rests on a definition of rectangle (the warrant), which the student makes explicit. The student correctly calculates slopes and side lengths, and also interprets the information correctly to show that the figure has the needed features (opposite, parallel sides and 4 right angles).

The response could be strengthened by the student more clearly identifying that opposite sides are congruent, but consecutive sides not congruent. The response could also be strengthened by explicitly stating that perpendicular line segments result when segments have slopes with values that are negative reciprocals.

Student 5

) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:
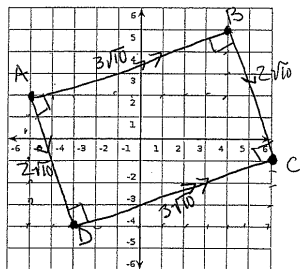
Parallelogram, Rectangle, Rhombus, Square, Trapezoid

se slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write

nathematical argument to justify your classification.

slope $\overline{AB} = \frac{3}{9} = \frac{1}{3}$

slope $\overline{DC} = \frac{3}{9} = \frac{1}{3}$ $\therefore$ $\overline{AB} \parallel \overline{DC}$

slope $\overline{AD} = \frac{-6}{2} = -3$

slope $\overline{BC} = \frac{-6}{2} = -3$ $\therefore$ $\overline{AD} \parallel \overline{BC}$

$\therefore \angle A = 90°$ $\overline{AB} \parallel \overline{DC}$
$\angle B = 90°$ $\overline{AD} \parallel \overline{BC}$
$\angle C = 90°$ $\overline{AB} \approx \overline{DC}$
$\angle D = 90°$ $\overline{AD} \approx \overline{BC}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$\overline{AB} = \sqrt{(4+5)^2 + (5-2)^2}$
$\overline{AB} = \sqrt{9^2 + 3^2}$
$\overline{AB} = \sqrt{81+9}$
$\overline{AB} = \sqrt{90}$
$\overline{AB} = 3\sqrt{10}$ $\therefore$ $\overline{DC} = 3\sqrt{10}$

Rectangle:
using the slope formula, I found that $\overline{AB}$ and $\overline{DC}$ are parallel because the have the same slope. $\overline{AD} \parallel \overline{BC}$ as well because they have the same slope. Their slopes are opposite and flipped over so all angles are 90°. However, since $\overline{BC} \not\approx \overline{AB}$ and $\overline{AD} \not\approx \overline{DC}$ all sides are not equal, making it a rectangle.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$\overline{BC} = \sqrt{(6-4)^2 + (-1-5)^2}$
$\overline{BC} = \sqrt{2^2 + (-6)^2}$
$\overline{BC} = \sqrt{4+36}$
$\overline{BC} = \sqrt{40}$
$\overline{BC} = 2\sqrt{10}$ $\therefore$ $\overline{AD} = 2\sqrt{10}$

Commentary

This student's argument was categorized as **HIGH quality**.

This student makes the claim of Rectangle and has evidence to support this: the student properly calculates slopes, infers angle measures from the slopes, and properly calculates side lengths, as well as interprets those calculations correctly. The student also identifies that not all sides are congruent, ruling out square.

Though the student doesn't state a definition for a rectangle (warrant), the student implies that if the following three criteria are met, then it is a rectangle: (1) opposite sides are parallel, (2) the angles are 90 degrees, and (3) if 2 pairs of sides have congruent length and not all 4 sides are congruent in length. [Note that (1) and (2) are needed to show the quadrilateral is a rectangle, and (3) shows that it is a rectangle that is also not a square.]

The response could be strengthened by the student more clearly identifying that not only are consecutive sides not congruent, but opposite sides *are* congruent, which is not explicitly stated, but implied by the comment that not all sides were congruent. The response could also be strengthened by a clear articulation of a definition of a rectangle.

Student 6

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$
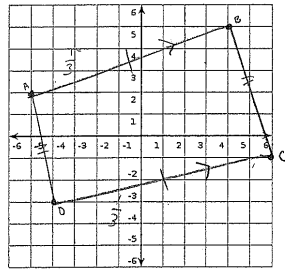
Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

Parallelogram, Rectangle, Rhombus, Square, Trapezoid

Use slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$$\frac{5-2}{4+^-5} \quad \frac{3}{9} \quad \frac{1}{3}$$

$$\frac{-4+^-1}{-3-6} \quad \frac{-3}{-9} \quad \frac{1}{3} \quad ||$$

$$\frac{-4-2}{-3+^-5} \quad \frac{-6}{-8} \quad ||$$

$$\frac{-1-5}{6-4} \quad \frac{-6}{2} \quad -3$$

the shape is a parallelogram because its opposite sides are both congruent & parallel. I found this using distance formula to get the length of the segment & the slope formula to see if they were parallel.

$$\sqrt{(-5+^-4)^2 + (2-5)^2}$$
$$\sqrt{(-9)^2 + (-3)^2}$$
$$\sqrt{81+9}$$

$$\sqrt{(6+^+3)^2 + (-1+^+4)^2}$$
$$\sqrt{(9)^2 + (3)^2}$$
$$\sqrt{81+9}$$
$$\sqrt{90}$$
$$\boxed{9.49}$$

$$\sqrt{(-5+^+3)^2 (2+^+4)^2} \quad \sqrt{(4-6)^2 + (5+1)^2}$$
$$\sqrt{(-2)^2 + (6)^2} \quad \sqrt{(-2)^2 + (6)^2}$$
$$\sqrt{4+36} \quad \sqrt{4+36}$$
$$\sqrt{40} \quad \sqrt{40}$$
$$\boxed{6.32} \cong \boxed{6.32}$$

$$\sqrt{(9)^2 + (3)^2}$$
$$\sqrt{81+9}$$
$$\sqrt{90}$$
$$\boxed{9.49}$$

Commentary

This student's argument was categorized as **ADEQUATE quality**.

This student makes the claim of Parallelogram and has evidence and warrant to support this claim. The student properly calculates slope and side lengths, and properly interprets this information to mean opposite sides are congruent and parallel. The student also offers a correct definition of parallelogram and shows that the necessary criteria are met. The argument that this is a *parallelogram* is complete. We did not factor in the wrong plot of point D since this was not used in the argument, and did not diminish the strength of the argument.

As the claim is for *the most specific classification possible,* the response could be extended by the student making explicit why the figure is not also a rhombus or any other shape listed. Furthermore, the response could be extended by recognizing the slope values indicate that adjacent sides are perpendicular creating 90 degree angles, ruling out parallelogram as the *most specific* classification of the quadrilateral.
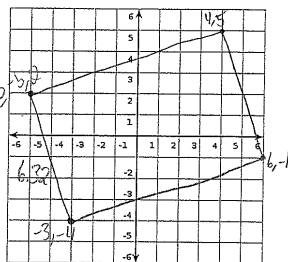
Student 8

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

This shape is a rectangle, It has equal opposite sides, and congruent opposite slopes,

$$d = \sqrt{(-4-2)^2 + (-3--5)^2}$$
$$d = \sqrt{(-6)^2 + (2)^2} = \sqrt{36+4} = \sqrt{40}$$
$$= 6.32$$

$$d = \sqrt{(4-6)^2 + (5--1)^2}$$
$$d = \sqrt{(-2)^2 + (-6)^2} = \sqrt{36+4} = \sqrt{40}$$
$$= 6.32$$

$\dfrac{5-2}{4--5} = \dfrac{3}{9} = \dfrac{1}{3} = \text{slope}$

$\dfrac{-1--4}{6--3} = \dfrac{3}{9} = \dfrac{1}{3} = \text{slope}$

$\Big\}= \text{length}$

$\dfrac{-4-2}{-3--5} = \dfrac{-6}{2} = \dfrac{-3}{1} = \text{slope}$

$\dfrac{-1-5}{6-4} = \dfrac{-6}{2} = \dfrac{-3}{1} = \text{slope}$

$\Big\}= \text{width}$

Commentary

This student's argument was categorized as **ADEQUATE quality**.

This student makes the claim of Rectangle and has evidence and an (implied) warrant to support this. (The implied warrant is a definition or theorem about rectangles: a rectangle is a figure with (a) equal opposite sides and (b) slopes that are negative reciprocal. Though accurate, it requires more information than needed: showing the quadrilateral has four right angles is sufficient to conclude it is a rectangle.) The student argues the figure is a rectangle, but does not argue that a rectangle is the *most specific* figure.

Note that the phrase "congruent opposite slopes" can be interpreted in different ways. One interpretation of "congruent opposite slopes" is that the two slopes have "opposite" values (negative reciprocals) which implies the figure is a rectangle (as interpreted above); the other interpretation is that the opposite sides have the same slope (which implies a parallelogram).

The response could be strengthened by the use of a more precise language in the warrant (e.g., revising "congruent opposite slopes"). The student could also make more clear how the second pair of opposite sides are known to be congruent, as these calculations are not shown. (To do this, the student could do the calculations or reference that having one pair of congruent sides and parallel slopes implies the other pair of opposite sides must be congruent.)

Finally, the response could be extended by providing evidence that the figure is a rectangle *and not a square,* which is needed to justify the rectangle as the *most specific* classification.

Student 1

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

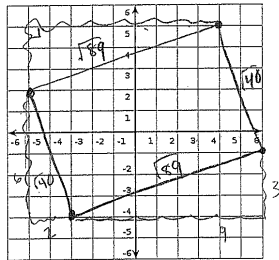**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

Slope A-D (-3)
Slope B-C -3
Slope AB 1/3
Slope C-D 1/3

Length A-D $2\sqrt{10}$
Length B-C $2\sqrt{10}$
Length A-B $2\sqrt{10}$
Length C-D $\sqrt{10}$

$\sqrt{(3--3)^2 + (4-2)^2}$
$\sqrt{4+36}$
$\sqrt{(6-4)^2 + (-1-5)^2}$
$\sqrt{4+36}$
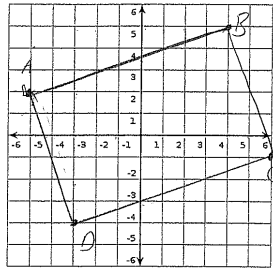
The shape is a square. Through mathematical calculations, all side lengths are reciprocal. Slope of A-D is $2\sqrt{10}$ and the slope of each line is its -3 and slope of C-D is 1/3. The side lengths are all equal and angles 90 due to slope. The characteristics of a square define this.

Commentary

This student's argument was categorized as **ADEQUATE quality**.

This student makes the claim of Square and has some evidence and warrant (a definition of a square) to support this. The student's definition of square is: side lengths are all equal and angles are 90 degrees. The student properly identifies the slopes, though no calculations are shown, and what the slopes mean in terms of angle measures. The student calculates the side length for one pair of opposite sides, and then asserts the other pair is also the same length. If that were true, the figure indeed would be a square. This assumed piece of evidence, however, leads to the incorrect claim.

The response could be strengthened by the student labeling their side length calculations. If all calculations were labeled properly, the student might have noticed they did not have enough side lengths to support their claim, thus potentially changing their claim.

The response could also be strengthened by including some indication of how the slope values were derived either from the coordinates provided or the diagram. Whether this needs to be included as part of the evidence, however, many depend on the agreed-upon requirements of an argument for a class.

Student 9

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

Parallelogram, (Rectangle,) Rhombus, Square, Trapezoid

Use slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$(-3,-4)(6,-1)$

$m = \left(\dfrac{6--3}{-1-4}\right) \quad \dfrac{9}{3} = \boxed{3}$

$m = (-5,2)(-4,-3)$

$\dfrac{-4-5}{-3-2} \quad \boxed{\dfrac{1}{-5}}$

$m = (4,5)(6,-1)$

$\dfrac{6-5}{-1-4} = \dfrac{1}{-5}$

This quadrilateral is a rectangle because there are 2 sets of congruent sides. The length is $\sqrt{89}$ and the width is $\sqrt{40}$. The slope of the length is 3 and the widths $-\frac{1}{5}$. The 2 sets of congruent sides (Length and width size difference) eliminates a rhombus and square since all sides are equal for them. Trapezoid is eliminated because it has 2 acute and 2 obtuse angles which this quad. lacks. Between rectangle + parallelogram it is rectangle because it appears to lack obtuse and acute angles.

$4 + 36 = \sqrt{40}$

$9 + 81 = \sqrt{89}$

May 2015

This student's argument was categorized as **borderline of ADEQUATE and LOW quality**.

This student makes the claim of Rectangle and uses an incomplete definition of a rectangle (warrant) to support this claim. The student says a rectangle is a quadrilateral if "there are 2 sets of congruent sides." The student has evidence to support this, showing sqrt 40 and sqrt 89 as the side lengths. The student also appropriately eliminates some of the other classifications, square and rhombus, "since all sides are equal for them." The student, however, makes assumptions about angle measurements of this quadrilateral without proper evidence to eliminate other figures (e.g., trapezoid and parallelogram). There are also calculation errors in the evidence (both in slope and side length).

The response could be strengthened by the student appropriately interpreting the slope calculations. The student calculates the values, but does not indicate what these values mean. (The student asserts the figure "appears to lack obtuse and acute angles." This is true, but not implied by the slope calculations.) If the student had properly interpreted the slope, then the miscalculation may have been caught or would have led to a different claim.

Student 2

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

Parallelogram, (Rectangle,) Rhombus, Square (Trapezoid)

Use slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$\sqrt{(4-(-5))^2 + (5-2)^2}$
$9^2 + 3^2$
$81 + 9$
$\sqrt{90}$

$\overline{AB} \cong \overline{DC}$

$\overline{AD} \cong \overline{BC}$

$\sqrt{(6--3)^2 + (-1-(-4))^2}$
$9^2 + 3^2$
$81 + 9$
$\sqrt{90}$
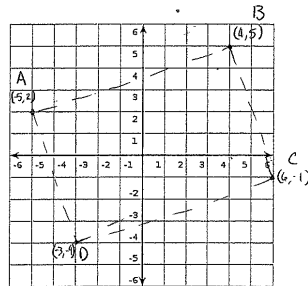
$m = \dfrac{5-2}{4--5} = \dfrac{3}{9} = \dfrac{1}{3}$ slope $AB \cong$ slope DC

$m = \dfrac{-1-(-4)}{6--3} = \dfrac{3}{9} = \dfrac{1}{3}$

$\sqrt{(-3--5)^2 + (-4-2)^2}$
$2^2 + (-6^2)$
$4 + 36$
$\sqrt{40}$

$m = \dfrac{2-(-4)}{-5-(-3)} = \dfrac{6}{-2}$ slope $AD \cong$ slope BC

$m = \dfrac{5-(-1)}{4-6} = \dfrac{6}{-2}$

$\sqrt{(6-4)^2 + (-1-5)^2}$
$2^2 + (-6^2)$
$4 + 36$
$\sqrt{40}$

The Shape is a rectangle

Commentary

This student's argument is classified as **LOW quality**.

This student makes the claim of Rectangle and has evidence to potentially support this claim. However, the lack of a clear warrant in this argument leads to a classification of low. It appears that some conclusions have been formed based on the evidence (e.g., congruence of pairs of sides) though there is no tie between the conclusions made and definition of a rectangle.

The response could be strengthened by the student providing a warrant, most likely in the form of a definition of a rectangle, to connect the evidence to the claim. The student has accurate calculations to show that this figure has (a) congruent opposite sides, with each pair being different lengths, and (b) sides that meet at right angles, as adjacent sides have slopes values that are negative reciprocals. Together (a) and (b) satisfy the definition of a rectangle, and so articulating this definition and the connection between the evidence and these two criteria would be sufficient to demonstrate the figure is a rectangle.
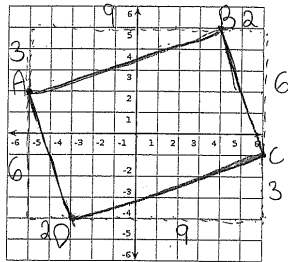
Student 7

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

Parallelogram, Rectangle, Rhombus, Square, Trapezoid

Use slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$$\frac{-1-5}{6-4} = \frac{-6}{2} = -3$$

$\overline{AD}$ and $\overline{BC}$ have the same slope

$$\frac{5-2}{4++5} \quad \frac{3}{9} = \frac{1}{3}$$

$$\frac{-4++1}{-3-6} \quad \frac{-3}{-9} = \frac{1}{3}$$

$\overline{AB}$ and $\overline{DC}$ have the same slope

$$9^2 + 3^2 = c^2 \quad \sqrt{811}$$
$$9^2 + 3^2 = c^2 \quad \sqrt{811}$$

$$6^2 + 2^2 = c^2 \quad \sqrt{1600}$$
$$6^2 + 2^2 = c^2$$

Using the pathagorem therom you can tell both are the same.

Student 4

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

Parallelogram, Rectangle, Rhombus, Square, Trapezoid

Use slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.
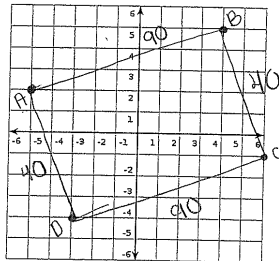
Obviously just by graphing the vertices you can see that this quadrilateral is a rectangle.
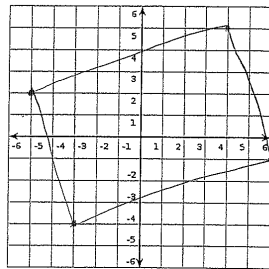
It has two sets of parallel lines, four 90° angles, yet 2 sides lengths are longer than the other set.

Student 10

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

Parallelogram, Rectangle, Rhombus, Square, Trapezoid

Use slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$$\frac{-4+2}{-3++5} = \frac{-6}{2} = -3$$

$$\frac{5+2}{4++5} = \frac{3}{9} = 3$$

$$\frac{-1+5}{6+4} = \frac{-6}{2} = -3$$

$$\frac{-1+-4}{6-+3} = \frac{3}{9} = 3$$

$AD = -3$
$AB = 3$
$BC = -3$
$DC = 3$

$$-3+-5 = 2^2 \quad -4 - 2 = -6^2$$
$$4 + 36 = 40 \quad AD$$

$$6 -4 = 2^2 \quad -1+5 = -6^2$$
$$4 + 36 = 40 \quad BC$$

$$4++5 = 9^2 \quad 5-2 = 3^2$$
$$81 + 9 = 90 \quad AB$$

$$-3+6 = -9^2 \quad -4++1 = -3^2$$
$$81 + 9 = 90 \quad OC$$

Key Connecting Sorting Packet to Argumentation Resource Packet

Student number (Sorting Packet)	Resource Packet Sample (Quality)
1	Adequate
2	Low
3	High
4	Low
5	High
6	Adequate
7	Low
8	Adequate
9	Border A/L
10	Low

Resource Packet Samples (Quality)	Student number (Sorting Packet)
High	3
High	5
Adequate	6
Adequate	8
Adequate	1
Adequate/Low	9
Low	2
Low	7
Low	4
Low	10