

Work Samples Classification and Commentaries

Task: Laura Says $\frac{1}{4}$ is Shaded, Grade 4

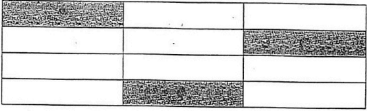
Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much “weight” people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional *discussions* and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

A Key linking the work samples from this ordered set with the sorting packet appears at the end of the document.

Student A

What fraction of the rectangle below is shaded?

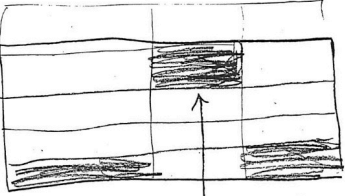
Think



Laura says that $\frac{1}{4}$ of the rectangle is shaded. Do you think she is correct?

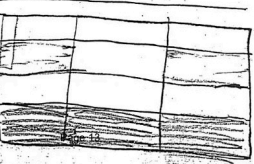
Yes because there is four groups and if you tape them there is $\frac{1}{4}$

Also $\frac{1}{4}$ is equal to $\frac{3}{12}$ Defend your answer.



Even though they are in a different order it's still equal to $\frac{1}{4}$

$$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$$


Commentary

This student's argument was categorized as **High Quality**.

Student A's claim is yes. The student identifies four groups (rows) and provides an argument to show that even though the shaded parts are in different rows, taken together ("tape") they ("still") equal $\frac{1}{4}$ of the rectangle. The student uses pictures and equations to show that $\frac{1}{4}$ is equivalent to $\frac{3}{12}$.

The student states that if you "tape" the shaded regions together (i.e., move them to the same row), they represent $\frac{1}{4}$ of the rectangle. This is supported by the two pictures. The student used a multiplication sentence showing that when you multiply $\frac{1}{4}$ by $\frac{3}{3}$, the result is an equivalent fraction: $\frac{3}{12}$. The student also used a series of equivalent fractions confirming the equivalence.

This argument is considered high quality even though the warrant is weak. The response could be strengthened by improving the precision of language (i.e., use composed instead of taped or four rows instead of four groups) and expanding upon the warrant of $\frac{1}{4} = \frac{3}{12}$, by explaining, for example, why multiplying $\frac{3}{3}$ creates an equivalent fraction. Notice also that the use of mathematical language is weak ("tape" and "groups"). Depending on the classroom norms, this could be considered a lower level argument.

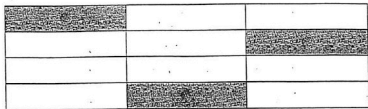
Argumentation Components

Claim	Evidence
The claim is stated: Yes.	The student represents visually and in writing that $\frac{3}{12} = \frac{1}{4}$. In writing, the student used a series of equivalent fractions and a multiplication sentence to show that when you multiply $\frac{1}{4}$ by $\frac{3}{3}$, the result is $\frac{3}{12}$. The student also says if you "tape" the shaded regions together, it equals $\frac{1}{4}$.
Warrants	Language & Computation
The student states that $\frac{1}{4} = \frac{3}{12}$ to support the claim. The student also linked the pictures with the statement that even though the shaded parts on each picture are in different order, each shaded region is equal to $\frac{1}{4}$ of the rectangle.	The use of "tape" and "four groups" may be an accepted norm in the class, however, in general, the mathematical language used here would be considered weak.

Student B

What fraction of the rectangle

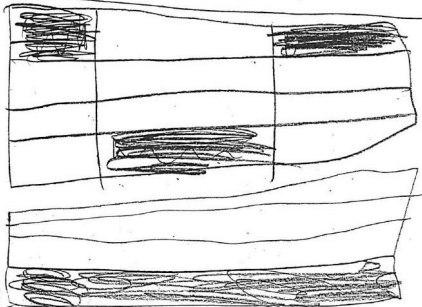
Think



Laura says that $\frac{1}{4}$ of the rectangle is shaded. Do you think she is correct?

Defend your answer.

Yes because $\frac{3}{12}$ is equivalent to $\frac{1}{4}$ because if you get rid of the vertical lines it equals $\frac{1}{4}$.



$\frac{1}{4}$

is = to

3/12 = 1/4

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Commentary

This student's argument was categorized as **High Quality**.

Student B's claim is yes. The student uses a model as evidence and an equation using cross multiplication to indicate reasoning that the three shaded parts represent $\frac{1}{4}$ of the whole. The student *implicitly* demonstrates understanding of how to interpret the value of a shaded region to represent equivalent fractions. Like Student A, Student B focuses on the rows (get rid of the vertical lines) and implies that moving all the shaded pieces down to the bottom row would create an area equivalent to $\frac{1}{4}$ but does not fully explain this rearrangement of the shaded pieces ("is = to").

The student uses adequate, although misspelled (e.g., ekwivelint), math language. Mathematical language could be expanded to explain the recomposing of the rectangle into four pieces instead of 12. It is recommended that spelling and symbolic issues be addressed at a later time and focus the assessment on the math argument.

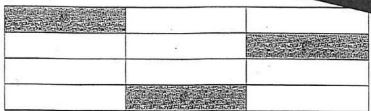
Argumentation Components

Claim	Evidence
The claim is stated: Yes.	The student uses a visual representation supported by a statement about how removing vertical lines creates four equal rows that will have one row shaded. The student uses cross-multiplication apparently as a check for equivalency.
Warrants	Language & Computation
The warrant is stated as “ $\frac{3}{12}$ is equivalent to $\frac{1}{4}$.” This warrant is connected to the evidence by stating that removing the vertical lines creates a region of four pieces with one shaded.	The student uses adequate, although misspelled, math language to create understanding of the process of rearranging the pieces into one row.

Student C

Think


What fraction of the rectangle



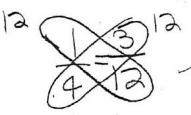
Laura says that $\frac{1}{4}$ of the rectangle is shaded. Do you think she is correct?

Defend your answer.

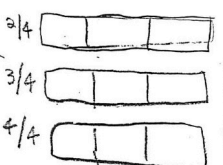
Yes, Laura is correct. She is because $\frac{1}{4}$ is equal to $\frac{3}{12}$.



$\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$ → This shows that the two fractions are equal



12



The tiles are just rearrange now it shows $\frac{1}{4}$, so Laura is correct

I have shown four ways that the fractions are equal. This shows that Laura is correct.

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Commentary

This student's argument was categorized as **High Quality**.

Student C's claim is, Yes, Laura is correct. Student C used multiple pictures, equations, and math language to show that $\frac{1}{4}$ is equivalent to $\frac{3}{12}$.

The student used a series of equivalent fractions and a cross multiplication equation to show that $\frac{1}{4} = \frac{3}{12}$. The student also used a model to show visually (although partially inaccurately) that rearranging the shaded parts into one row more clearly shows the shaded parts equal $\frac{1}{4}$ of the whole rectangle.

The warrant is made stronger by the statement, "This shows that the two fractions are equal." This statement connects the evidence to the claim.

The labeling of the bottom picture is inaccurate. The first bar is labeled $\frac{1}{4}$, however the shaded parts represent $\frac{3}{3}$, thus it is not clear what the label means. The next three bars are labeled $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$ respectively; however it is unclear how each of these represent those fractions. One can make different assumptions regarding what the thoughts of the student with this diagram, however, this is not accurately represented with the labels. The last sentence seems to imply that arguments need to be supported in different ways; however, this is not necessary for mathematical arguments.

This argument is considered high quality because the student shows understanding of how to interpret a model to represent a fraction that is not immediately obvious.

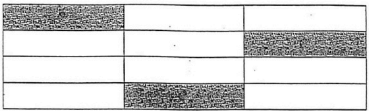
Argumentation Components

Claim	Evidence
The claim is stated: Yes, Laura is correct.	The student represents visually and in writing that $\frac{3}{12} = \frac{1}{4}$. In writing, the student used a series of equivalent fractions and a cross-multiplication diagram to show that the two fractions render the same value of 12. The student
Warrants	Language & Computation
The warrant is stated as " $\frac{1}{4}$ is equal to $\frac{3}{12}$." The warrant is supported in different ways that show the equivalence of the fractions.	The language used to support the argument is clear. As noted above, the labels on the bottom diagram are inaccurate.

Student D

What fraction of the rectangle bel

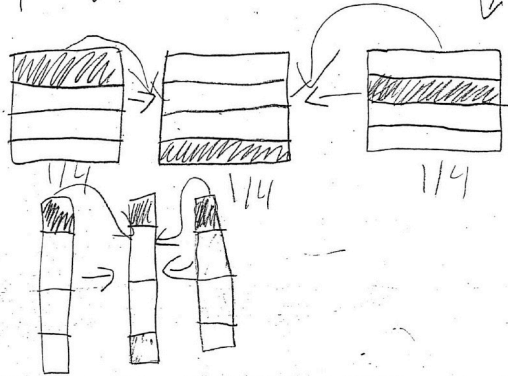
Think



Laura says that $\frac{1}{4}$ of the rectangle is shaded. Do you think she is correct?

Defend your answer.

Laura is correct because $\frac{3}{12}$ is equal to $\frac{1}{4}$. It shows me that Laura has 3 pieces of $\frac{1}{4}$ so Laura put the 3 pieces of $\frac{1}{4}$ together and she made $\frac{3}{12}$.



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Commentary

This student's argument was categorized as **Adequate Quality**.

Student D's claim is Laura is correct. The student uses a model (pictures and arrows) to show that joining all $\frac{1}{4}$ ths together results in the equivalent fraction $\frac{3}{12}$.

Unlike Student C, the model shows combining/rejoining of the 3 pieces to a whole in which $\frac{1}{4}$ is shaded. The model implies rejoining, but the language could be more precise in explaining this process.

The argument does not make it clear why it can be concluded that the rejoined model also represents $\frac{1}{4}$ of the rectangle.

The student uses cross multiplication to show that the two fractions are equal.

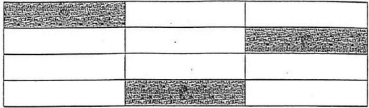
Argumentation Components

Claim	Evidence
The claim is stated: Laura is correct.	The student uses a model (pictures and arrows) to show that joining all $\frac{1}{4}$ ths together result in the equivalent fraction $\frac{3}{12}$. The student uses cross multiplication to verify equivalency.
Warrants	Language & Computation
Student's main support to the claim is that $\frac{3}{12}$ is equal to $\frac{1}{4}$. Student also links model to statement explaining that three $\frac{1}{4}$ ths, when considered together, represent $\frac{3}{12}$, but the explanation is incomplete.	The language and calculations used are correct; however, the argument would be stronger with an explanation of what the arrows between the diagram represent and appropriate warrants for that were offered.

Student E

Think

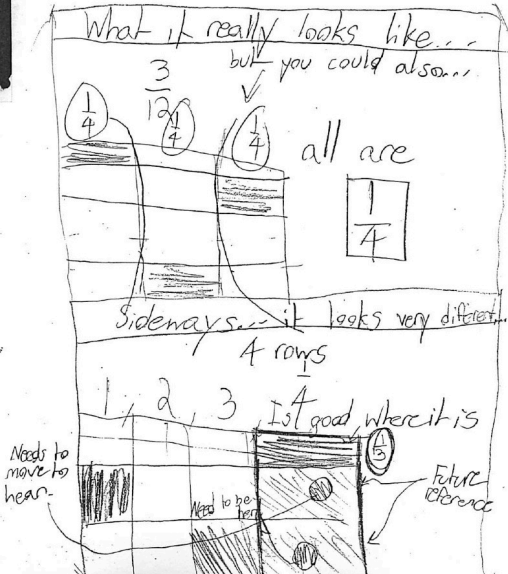
What fraction of the rectangle below is shaded?



Laura says that $\frac{1}{4}$ of the rectangle is shaded. Do you think she is correct? *Yes, I agree with Laura because of my work below.*

Defend your answer.

What it really looks like... but you could also...



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Commentary

This student's argument was categorized as **Adequate Quality**.

Student E's claim is yes. The student initially redrew the rectangle, showed that the shaded region represents $\frac{3}{12}$, and then indicated $\frac{1}{4}$ is in each one of the columns, "All are $\frac{1}{4}$." Student E then redrew the model sideways and showed that if you move two of the shaded parts so that all three shaded parts are in one column, it is easier to see the $\frac{1}{4}$ of the whole rectangle. By numbering the columns, it is implied that 1 of the four columns is shaded and therefore equal to $\frac{1}{4}$.

The student seems to understand that rearranging the shaded parts does not change the value of the fraction. The argument would be stronger if the student offered a warrant explicitly addressing the equivalency of $\frac{3}{12}$ and $\frac{1}{4}$ along with more precise mathematical vocabulary, such as whole, parts, and regions.

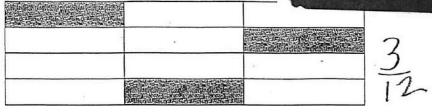
Argumentation Components

Claim	Evidence
The claim is stated: Yes.	Evidence is provided in the form of a new diagram in which the original has been redrawn by moving its shaded parts to show that the same diagram can also be seen as 1 column shaded out of 4.
Warrants	Language & Computation
The warrant used is that when the diagram represents the same fractions if looked at from a different perspective or moving the shaded parts to other places on the diagram.	The language and computations used is considered sufficient to follow the argument. None the less, the argument would be strengthened with more explanation and better vocabulary use (whole, parts, regions)

Student F

Think

What fraction of the rectangle bel



$\frac{3}{12}$

Laura says that $\frac{1}{4}$ of the rectangle is shaded. Do you think she is correct?

Defend your answer.

1 of the row

is right

It is

$\frac{1}{4}$ of the row

$\frac{1}{4}$ altogether

Page 4

Commentary

This student's argument was categorized as **Low Quality**.

Student F's claim is Laura is right. Student F identifies the original picture has $\frac{3}{12}$ shaded. The student also attempts to use a model to show that each column is $\frac{1}{4}$; however, there is no clear evidence of equality between $\frac{1}{4}$ and $\frac{3}{12}$. There are no warrants to link the beginnings of evidence to the claim.

Other explanations for the student's work may be possible, but would require making many inferences.

The student does not use precise vocabulary to clearly communicate the ideas that support the argument.

Argumentation Components

Claim	Evidence
The claim is stated: Laura is right.	Student identifies given model as $\frac{3}{12}$ and uses a model that shows $\frac{1}{4}$ th. However, it is difficult to interpret the connection between the student's model and the claim.
Warrants	Language & Computation
The explanation offered by the student to link the model to the claim is weak and hard to interpret: "1 of the row" and "4 altogether".	Vocabulary needs to be strengthened to make the argument clearer. For example, it is not clear how to interpret "altogether" in this fractions context.

Student G

What fraction of the rectangle below is shaded?

Think

Laura says that $\frac{1}{4}$ of the rectangle is shaded. Do you think she is correct?

Defend your answer.

Yes, I do or, thinks Laura is correct because there is three parts and each part has one shaded but if you rearrange them like this then it is still correct

Correct Laura

Correct

Page 1

Commentary

This student's argument was categorized as **Low Quality**.

Student G's claim is yes. Student G used a model to show that the value of the fraction does not change even though the arrangement is different.

Student G's response does not include precise language (e.g., does not explain what "parts" are). The models could be improved if drawn with mathematical accuracy (e.g., it is unclear if one of the models shows a 4×4 or 4×3 grid).

The argument would be stronger if the student provided the explanation that if you divide the whole into three smaller wholes, and each whole has $\frac{1}{3}$ shaded, then the combined shaded area will also represent $\frac{1}{3}$.

The student shows fair understanding but fails to make a fluent argument. The student does not use precise vocabulary to clearly communicate the ideas that support the argument.

Argumentation Components

Claim	Evidence
The claim is stated: Yes.	The student writes the statement "there is three parts and each part has one shaded..." and a visual to show that rearranging the parts of the whole does not change the value of the fractional part shaded.
Warrants	Language & Computation
The student does not use warrants to link the claim and the evidence.	The student's language lacks precision. For example, when writing "three parts" without being specific about which three parts. The student does not use mathematical vocabulary such as equivalent, equal to, or value.

Key Connecting Sorting Packet to Argumentation Resource Packet

Student number (Sorting Packet)	Resource Packet Sample
1	C (high)
2	E (adequate)
3	F (low)
4	A (high)
5	D (adequate)
6	B (high)
7	G (low)

Student number (Sorting Packet)	Resource Packet Sample (category)
4	A (high)
6	B (high)
1	C (high)
5	D (adequate)
2	E (adequate)
3	F (low)
7	G (low)