Fractional Parts of Candy Bars Problem (Grade 3)

STUDENT WORK SAMPLE ARGUMENTATION RESOURCE PACKET

This packet was produced as part of the Bridging Math Practices Math-Science Partnership Grant (2014 -2015).

The purpose of the packet is to help a) reveal what students can do with respect to generating an argument in response to mathematical questions, including the variety of their arguments; b) highlight features that should be considered when reviewing students' arguments, and c) identify what counts as a *quality* argument in light of the review criteria.

What is a mathematical argument?

A mathematical argument is

a sequence of statements and reasons given with the aim of demonstrating that a claim is true or false.

This links to the Connecticut Core Standards of Mathematical Practice #3, *construct viable arguments and critique the reasoning of others,* as well as other standards.

This resource packet is a product of work by participants in the UConn Bridging Math Practices Math-Science Partnership Grant, which included faculty and graduate students from the University of Connecticut's Neag School of Education and Department of Mathematics, and teachers and coaches from the Manchester Public Schools, Mansfield Public Schools, and Hartford Public Schools. This resource packet reflects significant contributions from Sarah Edwards, Myra Frosti, Kathleen Hackett, Shannon Harrington, Lisa Miner, Wendy Vincens.

Many thanks for all their insights and contributions! For more information about the grant, or for additional argumentation-related materials and resources, please see the project website: http://bridges.uconn.education.edu

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What is a high quality mathematical argument?

A high quality mathematical argument is an argument that shows that a claim must be true. It leaves little room to question. The chain of logic leads the reader to conclude that the author's claim is true.

What are the characteristics of a high quality argument? A high quality argument can be described by the following components and criteria:

Criteria	Description
1. A clearly stated **claim**	The claim is what is to be shown true or not true.
2. The necessary **evidence** to support the claim	Evidence can take the form of equations, tables, charts, diagrams, graphs, words, symbols, etc. It is one's "work" which provides the information to show something is true/false.
3. The necessary **warrants** to connect the evidence to the claim	Warrants can take the form of definitions, theorems, logical inferences, agreed upon facts. Warrants explain how the evidence is relevant for the claim, and collectively they chain the evidence together to show the claim is true or false.
4. **Language use and computations** are at a sufficient level of precision and accuracy	The language used and computations must be at a sufficient level of precision or accuracy to support the argument. Language use needs to be precise enough to communicate the ideas with sufficient clarity.

These criteria are helpful for discussions. It is important not to lose sight of the "big picture" however, and that is whether the argument offered shows that the claim is (or is not) true. This is the goal and purpose of a mathematical argument. You will see in many of these packets that students can approach an argumentation prompt from many different perspectives. It matters less *which* mathematical tools they use, and matters more whether their chain of reasoning compels the result.

In this packet you will find

1. A blank copy of the task, *Fractional Parts of a Candy Bar*, and a description of the context of the in which the student work samples included in this packet were produced.
2. A protocol that can help you and your colleagues discuss student work related to this task. The use of the protocol is optional.
3. Selected work samples on this task from 3rd-grade students in classes of teacher participants in the UConn Bridging Math Practices project to be used with the protocol.
4. Work Samples Classification and Commentaries: the student work samples ordered by whether they seem to be high, adequate, or low quality responses with respect to the criteria described on page 2 along with commentaries that support the classification. Among the samples are some that present a well-structured argument, but have important mathematical flaws, which prevent them from being classified as the highest quality.

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much "weight" people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional discussions and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries..

THE TASK

Comparing Fractional Parts of Candy Bars

- Olivia and Brett are eating candy bars. Olivia ate 1/3 of her candy bar and Brett ate ¼ of her candy bar.
  - – Brett said she ate the most.
  - – Olivia said she ate the most.
- Which student do you agree with and why?

CONTEXT: This task was given to third grade children who were working on comparing fractions. A common misconception is that ¼ is bigger than 1/3 because the number "4" is bigger than the number "3." Students used a variety of visuals to explain their thinking. They used an argumentation graphic organizer that was developed by UConn interns and BPCME project teachers. The purpose of the graphic organizer was to guide children in solving the problem using a claim, evidence and reasoning.

<<INSERT Protocol Here>>

Student 1

Peer Feedback Argument Frame

Problem:

Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

- Brett said he ate the most
- Olivia said she ate the most

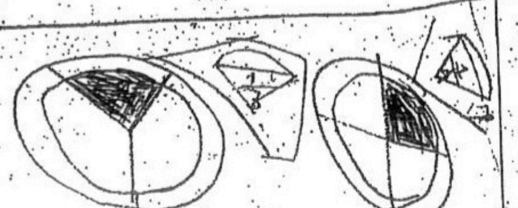
Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: I think olivia ate the most.

Evidence: Because the larger the denominator the smaller its fraction gets and 4 is larger than 3.

ex:



Reasoning: (I'm doing a diagram)

Student 2

Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

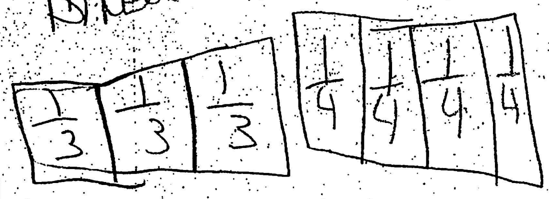
- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger then 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: Oliva is right

Evidence: $\frac{1}{3}$ has huge peaces $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{4}$ has smallen peaces



Reasoning: $\frac{1}{3}$ is biggen because its Fater $\frac{1}{3}$ wowld take two bites and $\frac{1}{4}$ takes 1 bite.

Student 3

Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

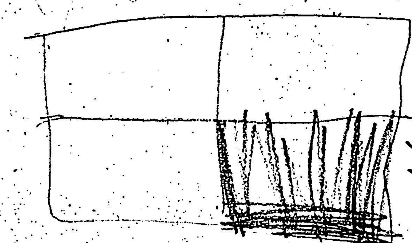
- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger then 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

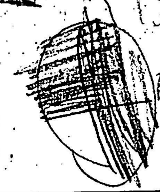
Claim: i think Olivia is right

Evidence: as we can see



you see that small

if compare



Reasoning: is better thane big small gives mor peices

Student 4

Peer Feedback Argument Frame

Problem:

Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

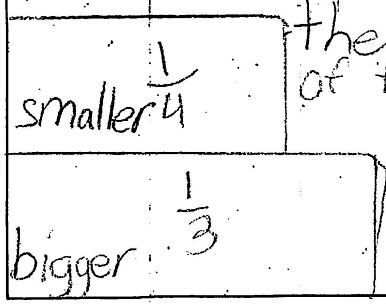
- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger then 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: Olivia has the biggest peace.

Evidence: She has the biggest part because when I was puting $\frac{1}{3}$ and $\frac{1}{4}$ together I new the smaller the denominator, the bigger size of the blocks.



Reasoning:

Student 5

Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

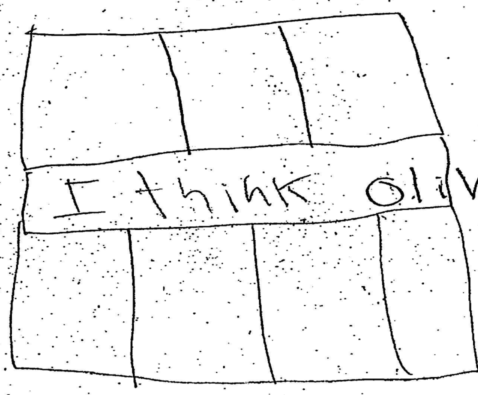
- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger then 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: I think olivia is right besides 3 is bigger than 4

Evidence:



I think olivia is right because the 3 peices are bigger than the four because with the four you cut a shape up

Reasoning: I think olivia is right because four parts are smaller because you would be sauishing a piece in and making other peices smaller

Student 6

Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

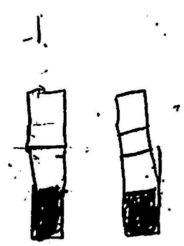
- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger then 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: Olivia is right because in fractons the lower dinomator the biger so $\frac{1}{3}$ is biger then $\frac{1}{4}$

Evidence:



you can see that $\frac{1}{3}$ is more then $\frac{1}{4}$

Reasoning: I the picture more for the $\frac{1}{3}$ is colerd then the $\frac{1}{4}$.

Student 7

Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger then 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: Olivia has the biggest peice

Evidence:

Olivia

$\frac{1}{3}$

is bigger

Brett

$\frac{1}{4}$

is Smaller

Reasoning:

because the bigger the number the smaller the size

$\frac{1}{4}$	$\frac{1}{3}$

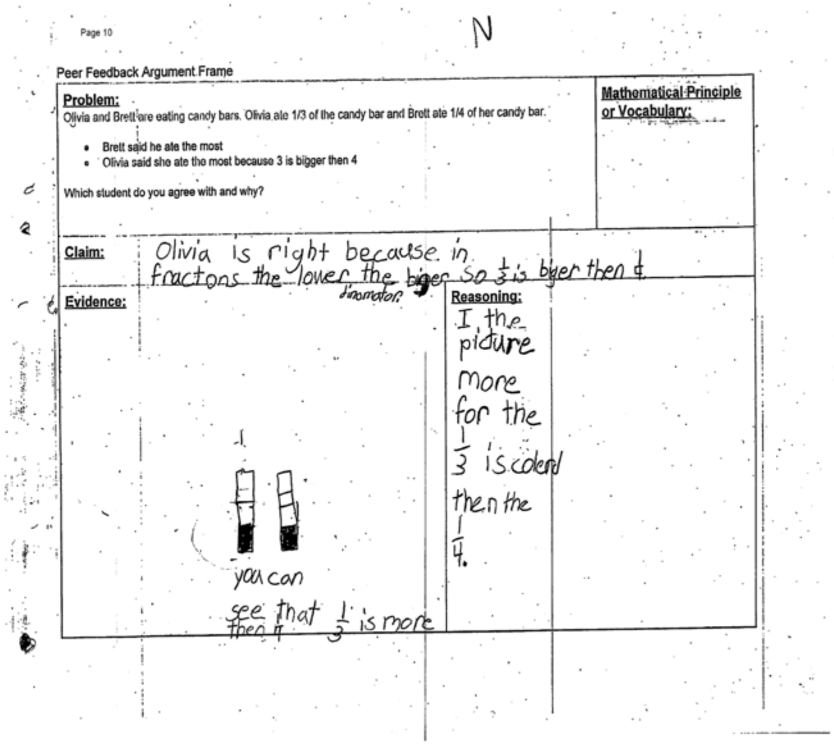
Work Samples Classification and Commentaries

Task: Comparing Fractional Parts of Candy Bars, Grade 3

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much "weight" people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional *discussions* and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

A Key linking the work samples from this ordered set with the sorting packet appears at the end of the document.

Student A



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Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger then 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: Olivia is right because in fractons the lower the dinomator the biger so 1/3 is biger then 1/4

Evidence: you can see that 1/3 is more then 1/4

Reasoning: I the picture more for the 1/3 is colerd then the 1/4.

Commentary

This student's argument was categorized as **High quality**.

The student's claim is that Olivia is correct.

The student draws two rectangles each representing *the whole and uses shading to show 1/3 and 1/4. The student then states that there is a larger shaded area for 1/3. The student provides reasoning that supports the visual evidence using appropriate mathematical vocabulary to compare the unit fractions by comparing their denominators.*

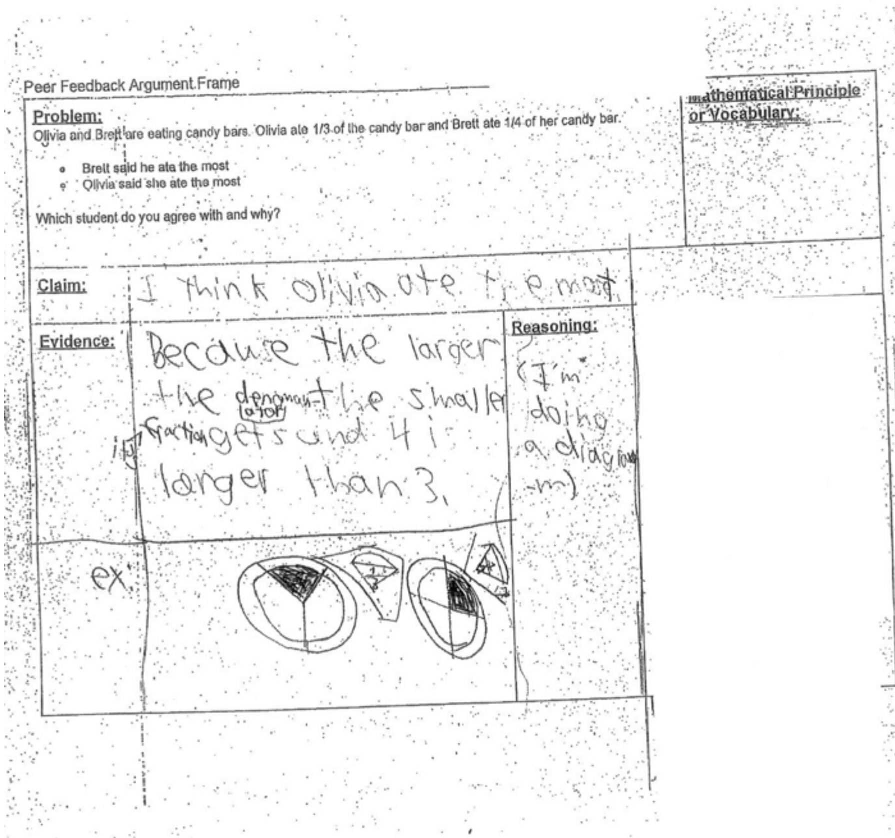
Although the pictures are not drawn to scale, it is understood that the student may not have had the proper measuring tool to show an accurate representation.

The argument could be strengthened if the student included labels in the diagram and referenced the fact that the numerators are the same.

Argumentation Components

Claim	**Evidence**
Olivia is right.	*The student compares two fraction bars using shading to show 1/3 is larger than 1/4.*
Warrants	**Language & Computation**
The student states that the lower the denominator the bigger the fraction and points out that the drawing shows a larger shaded area for 1/3.	*All mathematical computations and statements are correct with minor spelling errors. The drawings are partitioned and shaded correctly.*

Student B



Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

- Brett said he ate the most
- Olivia said she ate the most

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: I think olivia ate the most

Evidence: Because the larger the denominator the smaller fraction gets and 4 is larger than 3.

Reasoning: (I'm doing a diagram)

ex:

Commentary

This student's argument was categorized as **High quality**.

The student's claim is that Olivia is correct.

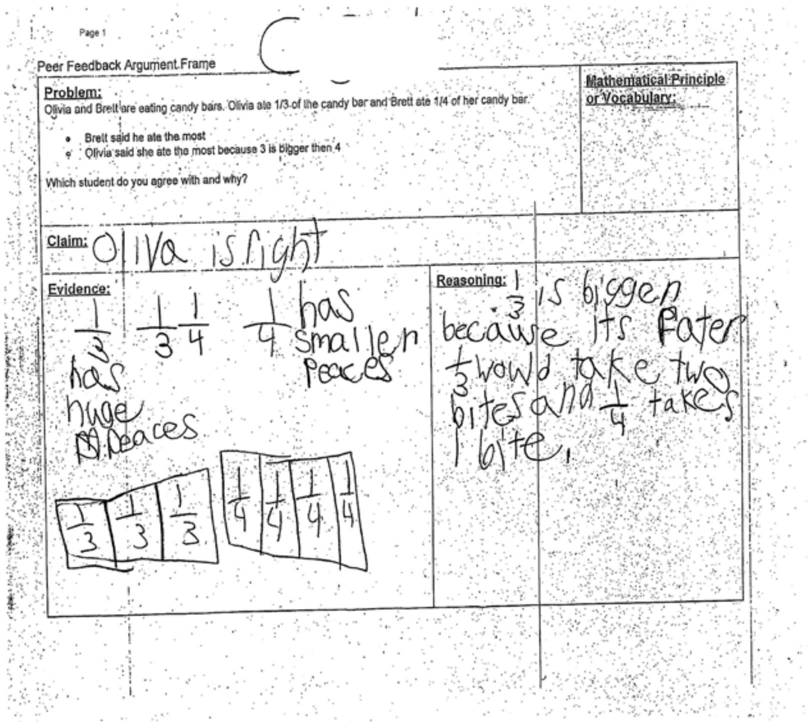
The student draws two circles representing the whole and uses shading to show 1/3 is larger than 1/4. The visual evidence is supported by the statement that the larger the denominator the smaller the fraction. The student uses clear language and appropriate mathematical vocabulary that shows the student is comparing fractions and not whole numbers.

The argument could be strengthened if the student included more labels in the diagram and referenced to the fact that the numerators are the same.

Argumentation Components

Claim	Evidence
Olivia ate the most.	*Two diagrams are drawn to compare 1/3 and 1/4.*
Warrants	**Language & Computation**
The student states that the greater the denominator the smaller the fraction.	*All drawings are correct and clear language and mathematics vocabulary is used to communicate ideas.*

Student C



Page 1

Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger then 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: Olivia is right

Evidence: 1/3 has huge peaces 1/3 1/4 1/4 has smallen peaces

1/3 1/3 1/3 1/4 1/4 1/4 1/4

Reasoning: 1/3 is biggen because its fater 1/3 would take two bites and 1/4 takes 1 bite.

Commentary

This student's argument was categorized as **Adequate quality.**

The student's claim is that Olivia is correct.

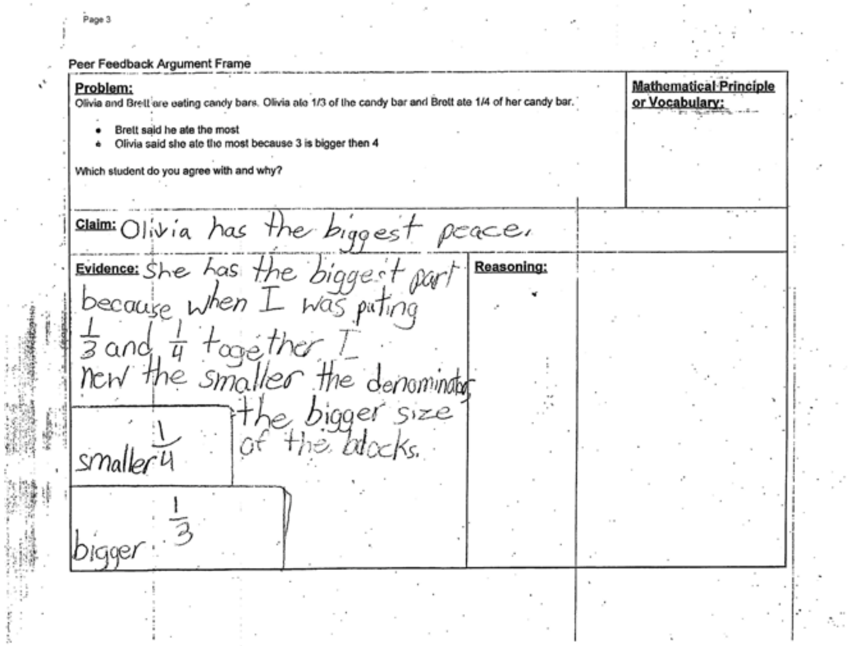
The student included two fraction bars to show that 1/3 is larger than 1/4. However the student did not use this evidence to appropriately justify the claim (e.g.: "1/3 would take 2 bites and 1/4 takes 1 bite").

There seems to be a misunderstanding about how many thirds would make a fourth (e.g.: "1/3 would take 2 bites and 1/4 takes 1 bite").

Argumentation Components

Claim	Evidence
Olivia is right	*The student uses two fraction bars and compares the sizes of the parts to show 1/3 is larger than 1/4.*
Warrants	**Language & Computation**
The student states that 1/3's are larger pieces than the 1/4's. However, these are not correctly interpreted to support the claim.	*Each fraction is represented correctly. The model is labeled correctly.*

Student D


Page 3

Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger then 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: Olivia has the biggest peace.

Evidence: She has the biggest part because when I was puting $\frac{1}{3}$ and $\frac{1}{4}$ together I new the smaller the denominator the bigger size of the blocks.

Reasoning:

Commentary

This student's argument was categorized as **Adequate quality.**

The student's claim is that Olivia is correct.

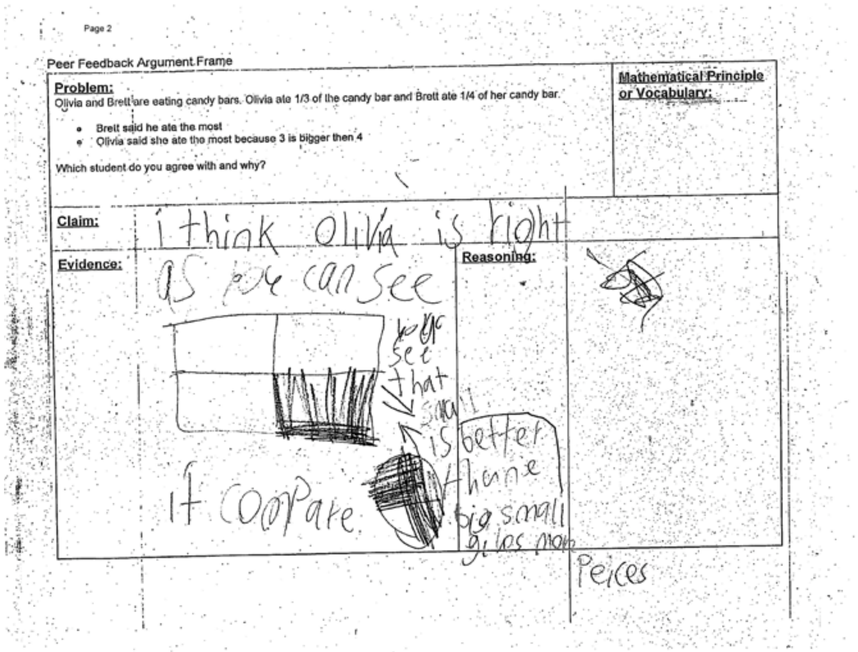
The student supports the claim with the statement that smaller denominators result in bigger unit fractions. The evidence is represented with an incomplete model showing fractional parts but the reference to the whole is not clear. In addition, there is no reference to the fact that the fractions have the same numerators, allowing the students to focus solely on the denominators.

The mathematical vocabulary could be improved (e.g. "blocks") to help make the reasoning clearer.

Argumentation Components

Claim	Evidence
Olivia has the biggest piece.	*Student shows an incomplete model showing fractional parts.*
Warrants	**Language & Computation**
Warrants are not correctly stated to support the statement "smaller denominators result in bigger size of the blocks"; reference to unit fractions would make this warrant clearer .	*Comparison of fractional parts is incomplete. The student uses some mathematical vocabulary to support the claim but the argument lacks clarity.*

Student E


Page 2

Peer Feedback Argument Frame

Problem: Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger then 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: i think Olivia is right

Evidence: as you can see

if compare

Reasoning: you see that small is better thane big small gives more Peices

Commentary

*This student's argument was categorized as **LOW quality.***

This is an example of a low quality argument because the evidence and warrant are unclear.

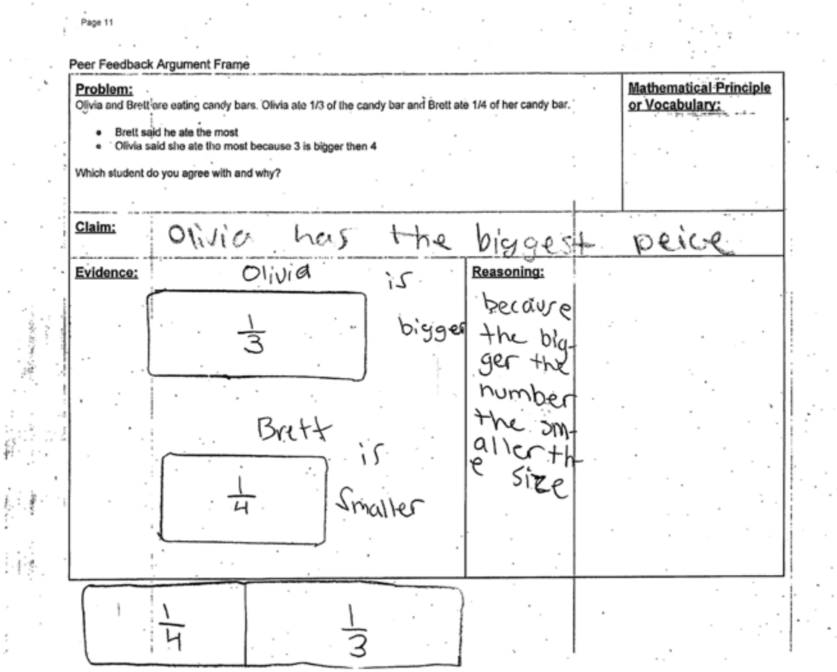
The work shows initial understanding that the focus of the problem is on the size of the unit fractions, that smaller denominators make smaller pieces, and that in this case "smaller is better" because smaller means larger. This thinking is not, however, demonstrated clearly in the diagrams or in the final statement "smaller gives more pieces".

The student correctly illustrates 1/4 of the rectangle. The student then draws a circle divided into 4 equal parts and shades 3 of the 4 parts apparently to illustrate 1/3. This model demonstrates a misunderstanding of how to represent 1/3. In addition, the student uses 2 different wholes to compare the fractions.

Argumentation Components

Claim	Evidence
I think Olivia is right.	*The student's evidence is represented in 2 diagrams.*
Warrants	**Language & Computation**
The student's warrant appears to be, "As you can see small is better than big." This does not adequately express an understanding of fractions required to support the claim.	*The student does not use appropriate mathematical vocabulary (e.g: denominator) to clearly express his or her thinking. "Parts" would be preferred over "pieces" to describe the fraction parts. The area models are not correctly drawn and are two different shapes. Additionally, the circle model is not an accurate representation of 1/3.*

Student F



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Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger then 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: Olivia has the biggest peice

Evidence: Olivia is bigger $\frac{1}{3}$ Brett is Smaller $\frac{1}{4}$

$\frac{1}{4}$ $\frac{1}{3}$

Reasoning: because the bigger the number the smaller the size

Commentary

This student's argument was categorized as **Low quality**.

The student's claim does not explicitly answer the question. The evidence provided has no referent whole. It seems that the student traced the actual fraction pieces which seems to link the evidence to the statement that "the bigger the number the smaller the size"; however, no clear connections are drawn between 1/3 and 1/4 to support the statement.

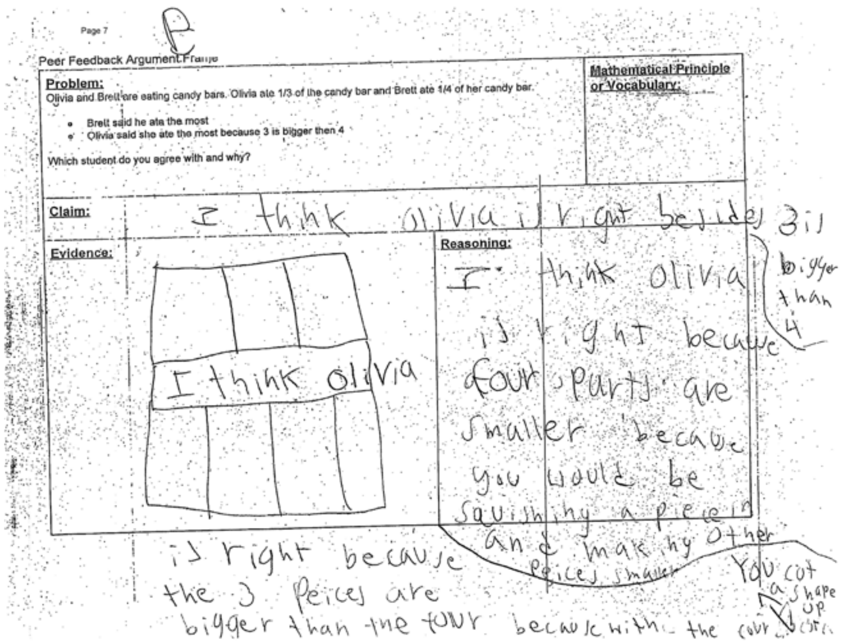
Note that it is unclear why the student placed the 1/4 and 1/3 fraction pieces side by side (horizontally) and this could be interpreted as a misconception of the parts to whole concept in fractions.

This argument could be strengthened by adding a part to whole relationship for 1/3 and 1/4 to a common whole and then comparing the size of each part. This could include an explanation of the smaller the denominator, the larger the part.

Argumentation Components

Claim	**Evidence**
Olivia has the biggest piece.	*The student shows models for 1/3 and 1/4 traced from the Fraction Strips to compare the sizes.*
Warrants	**Language & Computation**
A warrant is offered: "the bigger the number the smaller the size;" however it is not clearly connected to the evidence and it is insufficient to support the claim.	*The argument lacks appropriate mathematical vocabulary. "Parts" would be preferred over "pieces" to describe the fraction parts.* *The student did use comparative language but should have used "greater than" and "less than."*

Student G


Page 7

Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate 1/3 of the candy bar and Brett ate 1/4 of her candy bar.

- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger then 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: I think olivia is right besides 3 is bigger than 4

Evidence: I think olivia is right because the 3 peices are bigger than the four because with the

Reasoning: I think Olivia is right because four parts are smaller because you would be squishing a piece in and making other peices smaller

You cut a shape up

Commentary

This student's argument was difficult to categorize and is classified as a **Judgment Call**.

The student shows a diagram comparing fractional parts (1/4 and 1/3) of equal wholes. The student states that if a whole is divided into four parts, each part will be smaller than if the whole is divided into three parts because there are more parts. Both of these are the important mathematical concepts that the problem is eliciting. However, the student does not use these results to link it back to unit fractions and provide adequate support to the claim.

The statement "besides 3 is bigger than 4" is incorrect.

Argumentation Component

Claim	Evidence
I think Olivia is right	*The model shows fractional parts of thirds and fourths in relation to a whole.*
Warrants	**Language & Computation**
Some warrants are provided but they are not sufficient. e.g.if you put an extra piece in a small amount of space, each piece becomes smaller.	*The student does not use fraction notation or vocabulary. The fractions in the problem are not used in the explanation. The model is not labeled.*

Key Connecting Sorting Packet to Argumentation Resource Packet

Student number (Sorting Packet)	Resource Packet Sample
1	B (High)
2	C(adequate)
3	E(low)
4	D(adequate)
5	G (judgment call)
6	A (high)
7	F (low)

Student number (Sorting Packet)	Resource Packet Sample (category)
6	A (HIGH)
1	B (HIGH)
2	C (ADEQUATE)
4	D (ADEQUATE)
3	E (LOW)
7	F (LOW)
5	G (JUDGMENT CALL)