What is a mathematical argument?

A mathematical argument is

a sequence of statements and reasons given with the aim of demonstrating that a claim is true or false.

This links to the Connecticut Core Standards of Mathematical Practice #3, *construct viable arguments and critique the reasoning of others*, as well as other standards.

This resource packet is a product of work by participants in the UConn Bridging Math Practices Math-Science Partnership Grant, which included faculty and graduate students from the University of Connecticut’s Neag School of Education and Department of Mathematics, and teachers and coaches from the Manchester Public Schools, Mansfield Public Schools, and Hartford Public Schools. This resource packet reflects significant contributions from Sarah Edwards, Myra Frosti, Kathleen Hackett, Shannon Harrington, Lisa Miner, Wendy Vincens.

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What is a high quality mathematical argument?

A high quality mathematical argument is an argument that shows that a claim must be true. It leaves little room to question. The chain of logic leads the reader to conclude that the author’s claim is true.

What are the characteristics of a high quality argument? A high quality argument can be described by the following components and criteria:

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<tr>
<th>Criteria</th>
<th>Description</th>
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<td>1. A clearly stated claim</td>
<td>The claim is what is to be shown true or not true.</td>
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<td>2. The necessary evidence to support the claim</td>
<td>Evidence can take the form of equations, tables, charts, diagrams, graphs, words, symbols, etc. It is one’s “work” which provides the information to show something is true/false.</td>
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<td>3. The necessary warrants to connect the evidence to the claim</td>
<td>Warrants can take the form of definitions, theorems, logical inferences, agreed upon facts. Warrants explain how the evidence is relevant for the claim, and collectively they chain the evidence together to show the claim is true or false.</td>
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<td>4. Language use and computations are at a sufficient level of precision and accuracy</td>
<td>The language used and computations must be at a sufficient level of precision or accuracy to support the argument. Language use needs to be precise enough to communicate the ideas with sufficient clarity.</td>
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These criteria are helpful for discussions. It is important not to lose sight of the “big picture” however, and that is whether the argument offered shows that the claim is (or is not) true. This is the goal and purpose of a mathematical argument. You will see in many of these packets that students can approach an argumentation prompt from many different perspectives. It matters less which mathematical tools they use, and matters more whether their chain of reasoning compels the result.
In this packet you will find

1. A blank copy of the task, *Fractional Parts of a Candy Bar*, and a description of the context of the in which the student work samples included in this packet were produced.
2. A protocol that can help you and your colleagues discuss student work related to this task. The use of the protocol is optional.
3. Selected work samples on this task from 3rd-grade students in classes of teacher participants in the UConn Bridging Math Practices project to be used with the protocol.
4. Work Samples Classification and Commentaries: the student work samples ordered by whether they seem to be high, adequate, or low quality responses with respect to the criteria described on page 2 along with commentaries that support the classification. Among the samples are some that present a well-structured argument, but have important mathematical flaws, which prevent them from being classified as the highest quality.

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much “weight” people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional discussions and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

CONTEXT: This task was given to third grade children who were working on comparing fractions. A common misconception is that ¼ is bigger than 1/3 because the number “4” is bigger than the number “3.” Students used a variety of visuals to explain their thinking. They used an argumentation graphic organizer that was developed by UConn interns and BPCME project teachers. The purpose of the graphic organizer was to guide children in solving the problem using a claim, evidence and reasoning.