

Work Samples Classification and Commentaries

Task: Comparing Fractional Parts of Candy Bars, Grade 3

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much “weight” people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional *discussions* and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

A Key linking the work samples from this ordered set with the sorting packet appears at the end of the document.

Student A

Page 10

Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate $\frac{1}{3}$ of the candy bar and Brett ate $\frac{1}{4}$ of her candy bar.

- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger than 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: Olivia is right because in fractions the lower the bigger so $\frac{1}{3}$ is bigger than $\frac{1}{4}$

Evidence:

Reasoning: I the picture more for the $\frac{1}{3}$ is colored then the $\frac{1}{4}$.

you can see that $\frac{1}{3}$ is more than $\frac{1}{4}$



Commentary

This student's argument was categorized as **High quality**.

The student's claim is that Olivia is correct.

The student draws two rectangles each representing the whole and uses shading to show $\frac{1}{3}$ and $\frac{1}{4}$. The student then states that there is a larger shaded area for $\frac{1}{3}$. The student provides reasoning that supports the visual evidence using appropriate mathematical vocabulary to compare the unit fractions by comparing their denominators.

Although the pictures are not drawn to scale, it is understood that the student may not have had the proper measuring tool to show an accurate representation.

The argument could be strengthened if the student included labels in the diagram and referenced the fact that the numerators are the same.

Argumentation Components

Claim	Evidence
Olivia is right.	The student compares two fraction bars using shading to show $\frac{1}{3}$ is larger than $\frac{1}{4}$.
Warrants	Language & Computation
The student states that the lower the denominator the bigger the fraction and points out that the drawing shows a larger shaded area for $\frac{1}{3}$.	All mathematical computations and statements are correct with minor spelling errors. The drawings are partitioned and shaded correctly.

Student B

Peer Feedback Argument Frame

Problem:

Olivia and Brett are eating candy bars. Olivia ate $\frac{1}{3}$ of the candy bar and Brett ate $\frac{1}{4}$ of her candy bar.

- Brett said he ate the most
- Olivia said she ate the most

Which student do you agree with and why?

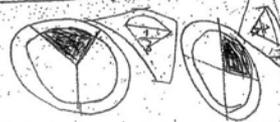
Mathematical Principle or Vocabulary

Claim: I think Olivia ate the most

Evidence: Because the larger the denominator the smaller the fraction gets and 4 is larger than 3.

Reasoning: (I'm doing a diagram)

ex



Commentary

This student's argument was categorized as **High quality**.

The student's claim is that Olivia is correct.

The student draws two circles representing the whole and uses shading to show $\frac{1}{3}$ is larger than $\frac{1}{4}$. The visual evidence is supported by the statement that the larger the denominator the smaller the fraction. The student uses clear language and appropriate mathematical vocabulary that shows the student is comparing fractions and not whole numbers.

The argument could be strengthened if the student included more labels in the diagram and referenced to the fact that the numerators are the same.

Argumentation Components

Claim	Evidence
Olivia ate the most.	Two diagrams are drawn to compare $\frac{1}{3}$ and $\frac{1}{4}$.
Warrants	Language & Computation
<i>The student states that the greater the denominator the smaller the fraction.</i>	<i>All drawings are correct and clear language and mathematics vocabulary is used to communicate ideas.</i>

Student C

Page 1

Peer Feedback Argument Frame

Problem:
Olivia and Brett are eating candy bars. Olivia ate $\frac{1}{3}$ of the candy bar and Brett ate $\frac{1}{4}$ of her candy bar.

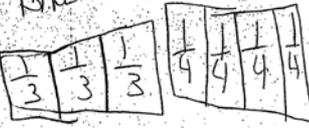
- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger than 4

Which student do you agree with and why?

Claim: Olivia is right

Evidence:
 $\frac{1}{3}$ has huge pieces.
 $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{4}$ has smaller pieces

Reasoning: $\frac{1}{3}$ is bigger because its denominator would take two bites and $\frac{1}{4}$ takes 1 bite.



Commentary

This student's argument was categorized as **Adequate quality**.

The student's claim is that Olivia is correct.

The student included two fraction bars to show that $\frac{1}{3}$ is larger than $\frac{1}{4}$. However the student did not use this evidence to appropriately justify the claim (e.g.: " $\frac{1}{3}$ would take 2 bites and $\frac{1}{4}$ takes 1 bite").

There seems to be a misunderstanding about how many thirds would make a fourth (e.g.: " $\frac{1}{3}$ would take 2 bites and $\frac{1}{4}$ takes 1 bite").

Argumentation Components

Claim	Evidence
Olivia is right	The student uses two fraction bars and compares the sizes of the parts to show $\frac{1}{3}$ is larger than $\frac{1}{4}$.
Warrants	Language & Computation
The student states that $\frac{1}{3}$'s are larger pieces than the $\frac{1}{4}$'s. However, these are not correctly interpreted to support the claim.	Each fraction is represented correctly. The model is labeled correctly.

Student D

Page 3

Peer Feedback Argument Frame

<p>Problem: Olivia and Brett are eating candy bars. Olivia ate $\frac{1}{3}$ of the candy bar and Brett ate $\frac{1}{4}$ of her candy bar.</p> <ul style="list-style-type: none"> Brett said he ate the most Olivia said she ate the most because 3 is bigger than 4 <p>Which student do you agree with and why?</p>		<p>Mathematical Principle or Vocabulary:</p>
<p>Claim: Olivia has the biggest piece.</p>		
<p>Evidence: She has the biggest part because when I was putting $\frac{1}{3}$ and $\frac{1}{4}$ together I now the smaller the denominator the bigger size of the blocks.</p>	<p>Reasoning:</p>	

Commentary

This student's argument was categorized as **Adequate quality**.

The student's claim is that Olivia is correct.

The student supports the claim with the statement that smaller denominators result in bigger unit fractions. The evidence is represented with an incomplete model showing fractional parts but the reference to the whole is not clear. In addition, there is no reference to the fact that the fractions have the same numerators, allowing the students to focus solely on the denominators.

The mathematical vocabulary could be improved (e.g. "blocks") to help make the reasoning clearer.

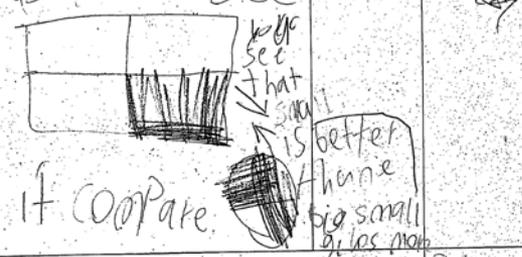
Argumentation Components

Claim	Evidence
Olivia has the biggest piece.	Student shows an incomplete model showing fractional parts.
Warrants	Language & Computation
Warrants are not correctly stated to support the statement "smaller denominators result in bigger size of the blocks"; reference to unit fractions would make this warrant clearer .	Comparison of fractional parts is incomplete. The student uses some mathematical vocabulary to support the claim but the argument lacks clarity.

Student E

Page 2

Peer Feedback Argument Frame

<p>Problem: Olivia and Brett are eating candy bars. Olivia ate $\frac{1}{3}$ of the candy bar and Brett ate $\frac{1}{4}$ of her candy bar.</p> <ul style="list-style-type: none"> Brett said he ate the most Olivia said she ate the most because 3 is bigger than 4 <p>Which student do you agree with and why?</p>	<p>Mathematical Principles or Vocabulary:</p>
<p>Claim: I think Olivia is right</p> <p>Evidence: As we can see</p>  <p>if compare.</p>	<p>Reasoning:</p> <p>that is better than here big small gives more pieces</p>

Commentary

This student's argument was categorized as **LOW quality**.

This is an example of a low quality argument because the evidence and warrant are unclear.

The work shows initial understanding that the focus of the problem is on the size of the unit fractions, that smaller denominators make smaller pieces, and that in this case "smaller is better" because smaller means larger. This thinking is not, however, demonstrated clearly in the diagrams or in the final statement "smaller gives more pieces".

The student correctly illustrates $\frac{1}{4}$ of the rectangle. The student then draws a circle divided into 4 equal parts and shades 3 of the 4 parts apparently to illustrate $\frac{1}{3}$. This model demonstrates a misunderstanding of how to represent $\frac{1}{3}$. In addition, the student uses 2 different wholes to compare the fractions.

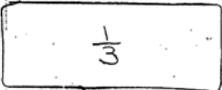
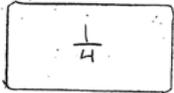
Argumentation Components

Claim	Evidence
<p>I think Olivia is right.</p>	<p>The student's evidence is represented in 2 diagrams.</p>
Warrants	Language & Computation
<p>The student's warrant appears to be, "As you can see small is better than big." This does not adequately express an understanding of fractions required to support the claim.</p>	<p>The student does not use appropriate mathematical vocabulary (e.g: denominator) to clearly express his or her thinking. "Parts" would be preferred over "pieces" to describe the fraction parts. The area models are not correctly drawn and are two different shapes. Additionally, the circle model is not an accurate representation of $\frac{1}{3}$.</p>

Student F

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Peer Feedback Argument Frame

Problem: Olivia and Brett are eating candy bars. Olivia ate $\frac{1}{3}$ of the candy bar and Brett ate $\frac{1}{4}$ of her candy bar.		Mathematical Principle or Vocabulary:
<ul style="list-style-type: none"> Brett said he ate the most Olivia said she ate the most because 3 is bigger than 4 		
Which student do you agree with and why?		
Claim: Olivia has the biggest peice		
Evidence: Olivia is bigger  Brett is smaller 	Reasoning: because the bigger the number the smaller the size	
		

Commentary

This student's argument was categorized as **Low quality**.

The student's claim does not explicitly answer the question. The evidence provided has no referent whole. It seems that the student traced the actual fraction pieces which seems to link the evidence to the statement that "the bigger the number the smaller the size"; however, no clear connections are drawn between $\frac{1}{3}$ and $\frac{1}{4}$ to support the statement.

Note that it is unclear why the student placed the $\frac{1}{4}$ and $\frac{1}{3}$ fraction pieces side by side (horizontally) and this could be interpreted as a misconception of the parts to whole concept in fractions.

This argument could be strengthened by adding a part to whole relationship for $\frac{1}{3}$ and $\frac{1}{4}$ to a common whole and then comparing the size of each part. This could include an explanation of the smaller the denominator, the larger the part.

Argumentation Components

Claim	Evidence
Olivia has the biggest piece.	The student shows models for $\frac{1}{3}$ and $\frac{1}{4}$ traced from the Fraction Strips to compare the sizes.
Warrants	Language & Computation
A warrant is offered: "the bigger the number the smaller the size;" however it is not clearly connected to the evidence and it is insufficient to support the claim.	The argument lacks appropriate mathematical vocabulary. "Parts" would be preferred over "pieces" to describe the fraction parts. The student did use comparative language but should have used "greater than" and "less than."

Student G

Commentary

This student's argument was difficult to categorize and is classified as a **Judgment Call**.

The student shows a diagram comparing fractional parts ($\frac{1}{4}$ and $\frac{1}{3}$) of equal wholes. The student states that if a whole is divided into four parts, each part will be smaller than if the whole is divided into three parts because there are more parts. Both of these are the important mathematical concepts that the problem is eliciting. However, the student does not use these results to link it back to unit fractions and provide adequate support to the claim.

The statement "besides 3 is bigger than 4" is incorrect.

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Peer Feedback Argument Form

Problem: Olivia and Brett are eating candy bars. Olivia ate $\frac{1}{3}$ of the candy bar and Brett ate $\frac{1}{4}$ of her candy bar.

- Brett said he ate the most
- Olivia said she ate the most because 3 is bigger than 4

Which student do you agree with and why?

Mathematical Principle or Vocabulary:

Claim: I think olivia is right beside 3 is bigger than 4

Evidence: 

Reasoning: I think olivia is right because four parts are smaller because you would be squishing a piece and making other pieces smaller. You cut a shape up

is right because the 3 pieces are bigger than the 4 because with the 4

Argumentation Component

Claim	Evidence
I think Olivia is right	The model shows fractional parts of thirds and fourths in relation to a whole.
Warrants	Language & Computation
Some warrants are provided but they are not sufficient. e.g. if you put an extra piece in a small amount of space, each piece becomes smaller.	The student does not use fraction notation or vocabulary. The fractions in the problem are not used in the explanation. The model is not labeled.

Key Connecting Sorting Packet to Argumentation Resource Packet

Student number (Sorting Packet)	Resource Packet Sample
1	B (High)
2	C(adequate)
3	E(low)
4	D(adequate)
5	G (judgment call)
6	A (high)
7	F (low)

Student number (Sorting Packet)	Resource Packet Sample (category)
6	A (HIGH)
1	B (HIGH)
2	C (ADEQUATE)
4	D (ADEQUATE)
3	E (LOW)
7	F (LOW)
5	G (JUDGMENT CALL)