# What is Mathematical Argumentation?



ATOMIC December 8, 2015 Session 4 Madelyn Colonnese (UConn), Sarah Brown (Manchester PS) Megan Staples (UConn) Jillian Cavanna (Michigan State University)

Presentation based on the Bridging Practices Across Mathematics Educators (BPCME) Project 2014-2015 funded by the CT State Department of Education, Math-Science Partnership Grant Program.

### Behavior Progression for Partner Argumentation

#### Phase One

- Facing each other, hands and legs quiet.
- Looking at each other.
- One person shares their claim, evidence and warrant and the other person listens.
- The other person can say or write exactly what the other person has just told them.
- Repeat with the other person

#### Phase Two

- One person shares their claim, evidence and warrant and the other person listens.
- The other person can say or write in their own words and ask questions what the
  other person has just told them.
- · Repeat with the other person.

### Phase Three

- One person shares their idea and the other person listens.
- The other person agrees or disagrees with the other person's claim, evidence and/or warrant and explains why. (Concentrate on agreeing or disagreeing with the argument, NOT the person.)
- Repeat with the other person.

### Phase Four

- Each person shares their claim, evidence, and warrant.
- They ask each other clarifying questions.
- They ask each other to explain their evidence in a different way.
- They agree or disagree with each other and state why using their evidence.
- They build upon what they are learning from their partner's argument and improve or change their own argument.

### Questions that teachers can ask to promote students' Argumentation

Can you explain?

What happened before?

What would happen if you used this number?

What would change if...

What does that look like?

What happened after?

Can you tell me why...

Tell me more.

Show me where...

What could you add to strengthen this part?

How would that work?

How does this fit?

Why did you use that equation?

So you are saying?

### $6 \times 5$ Argument Lena says that $6 \times 5 = 26$ . Do you agree or disagree with Lena?

Claim (I agree) (I disagree)	I agree/disagree with Lena's argument that 6 x 5 = 26.
<ul> <li>Evidence</li> <li>Draw a picture.</li> <li>Make a equation.</li> <li>Use a number line.</li> <li>Use place value blocks.</li> </ul>	
Warrant (Connects the evidence to the claim.)	

Name_	Date 11/13/15
	6 X 5 Argument & S

Lena says that  $6 \times 5 = 26$ . Do you agree or disagree with Lena?

Claim (I agree) (I disagree)	I discorree that 6x524
Evidence  Draw a picture.  Make a equation  Use a number line.  Use oplace	15, 15, 25, 36, 56, 56, 56, 56, 56, 56, 56, 56, 56, 5
value blocks.	you were going with this, can you go to your description to
Warrant (Connects the evidence to the claim)	T disagree that 6x5=26 Help me ordership the condensation of the c

Name	Date VOVE mber 3th
Lena says tha	6 x 5 Argument at 6 x 5 = 26. Do you agree or disagree with Lena?
Claim (Lagree) (I disagree)	I disagree that
Evidence  Draw a picture.  Make a equation.  Use a number line.  Use place value blocks.	Goroups with 5
Warrant (Connects the evidence to the claim)	Toisagroe with lengor  Toyousxip count by  Syou gos, loist, Loid,  30.5030is the Anser So.

Name	Date	Olember 17
	Even/Odd Argument	2015

Turner says that an odd number is always any number(n) +1. Do you agree or disagree with Turner?

Claim (I agree) (I disagree)  Evidence  Draw a picture.  Make a equation.  Use a number line.  Use place value blocks.	# 1 = 5   1 + 10 = 10 E 0 0   F 0 0 2 + 1 = 3   1 + 16 = 17 E 0 0   0 E 0 1,000,000 + 1 = 1,000,000 E 0 0   1 = 1,000,000
Warrant (Connects the evidence to the claim.)	Flagree Because I showed it with a picture or acowashoin, recom

	:	D-4-11 17 15
Name		Date   - 17-15
, (0,110		

### Even/Odd Argument

Turner says that an odd number is always any number(n) +1. Do you agree or disagree with Turner?

Claim (I agree) (I disagree)	I disaggree that an odd number is always any number +1.
Evidence Draw a picture. Make a equation. Use a number line. Use place value blocks.	0000000+0=8  5+1=6  7+1=8  принципана=6  горинана=8
Warrant (Connects the evidence to the claim.)	I found that an old number +1 is always an even number because after an old number is

# A Few More Resources for Integrating Argumentation into your Mathematics Class!



The Bridging Practices Among Connecticut Mathematics Educators (BPCME) grant supported teachers from Hartford, Manchester, and Mansfield to understand and implement the third Mathematical Practice (MP3) of the Connecticut Core Standards, "Construct viable arguments and critique the reasoning of others". As a result of this work, the following products are available to be shared with teachers across the state of Connecticut.

### **Argumentation Resource Packets (ARPs)**

You can access the resource packets at:

http://bridges.education.uconn.edu/argumentation-resource-packets/

An Argumentation Resource Packet is a set of student work samples from classroom implementation of an argumentation task, along with commentaries about the student arguments that were developed by project team members and participants.

What is the structure of these resources?

Each Argumentation Resource Packet contains the following:

- 1. Introduction to Argumentation: criteria for a high quality argument
- 2 The Task
- 3. The Sorting Packet is a set of student work (~5-10 samples in total) from a classroom implementation of an argumentation task that shows a range of approaches to the argument and a range of proficiencies with the quality/completeness of the argument and expressing an argument.
- 4. A "Tuning" protocol supports the use of the Sorting Packets. Teachers can use this protocol to guide their categorization and discussion of the student work before looking at the project-generated commentaries.
- 5. "Student Work with Commentaries" are commentaries that accompany each student work sample. There are two types: i) a more holistic description that identifies the agreed-upon categorization (high quality, adequate, low), describes the approach, and notes important strengths and areas for improvement; and ii) targeted commentary on the four specific criteria/components (claim, warrants, evidence, and language/computations).

### **Task Repository**

You can access the task repository at: http://bridges.education.uconn.edu/repository/

The task repository contains tasks and tools focused on argumentation developed by Bridging Math Practices participants. Each task includes a brief description and is categorized by grade, math topic, domain and title. At the present time, the **Task Repository** includes over 200 tasks and tools for grades 1-6 and high school.

#### Resources for Integrating Written Argumentation into your Mathematics Classroom

You can access additional articles at:

http://bridges.education.uconn.edu/academic-year-workshop/

On the write path: Improving communication in an elementary mathematics classroom: The authors of this article, Little and Anderson, are two fourth-grade teachers who became interested in supporting their students' written and oral communication after they observed their students' difficulty in communicating their thinking when problem solving. This was also evident on standardized mathematics test scores where problem solving was the greatest area of weakness. Little and Anderson explain in this article how they assessed students' challenges with writing through a survey and observation. They describe the supports put in place to help different students and share their reflection on this process.

Little, D. M., & Anderson, M. A. (2004). On the write path: Improving

communication in an elementary mathematics classroom. Teaching children mathematics, 10(9), 468-472.

**Advice for Mathematical Argumentation:** Three middle school teachers engage students in argumentation by *telling* and *showing* them how to argue. A mathematical graphic organizer with three sections (Conjecture, Justification, and Conclusion) was used to help students organize their mathematical arguments.

Knudsen, J. & Lara-Meloy, T. (April 2014). Advice for mathematical argumentation.

MTMS 19(8), 494-500.

**Developing Students' Capacity for Constructing Proofs through Discourse:** By analyzing various ways students construct proofs, the authors provide recommendations on how teacher discourse can enhance skills involved in proof.

Stylianou, D. A., & Blanton, M. L. (2011). Developing students' capacity for

constructing proofs through discourse. The Mathematics Teacher, 105(2), 140–145.

These resources were developed by members of the UConn Bridging Math Practices Math and Science Partnership Grant (MSP), funded by the Connecticut State Department of Education, 2015-2015.

# **Argument Rubric**

Argument	1	2	3	4	5
Component	Does not meet expectations	Needs improvement	Approaching expectations	Meets expectations	Exceeds expectations
Claim (x2)	I did not state a claim or take a stance, nor did I imply agreement or disagreement.	I implied agreement or disagreement but I didn't clearly state a claim.	I stated a claim but did not clearly agree or disagree.	I stated a claim and took a stance in agreement or disagreement.	I stated a claim and took a stance that agrees with my argument.
Basis of Argument (x4)	Does not state a principle or definition.	Implies a principle or definition.	States a principle or definition.	Clearly states a principle or definition that is widely regarded as valid.	Clearly states a principle or definition and cites the source.
Evidence (Grounds) (x5)	I did not use any evidence to support my claim.	I used limited evidence and it does not clearly or fully support my claim.	I used some evidence but it is not detailed enough to fully support my claim.	I used appropriate and detailed evidence to fully support my claim Expressions - Equations - Pictures - Graphs	I used appropriate and detailed evidence to fully support my claim. Verified solution with 2 <sup>nd</sup> strategy from previous column.
Reasoning (x5)	I did not include statements in order to describe my evidence.	I included statements which do not fully describe evidence.	I included a series of statements which describe and explain evidence.	I clearly justified my stance by including clear statements to lead the reader from the claim through the evidence.	I clearly justified my stance by including clear, logical and organized statements to lead the reader from the claim through the evidence to a conclusion.
Accuracy (x3)	I did not include any evidence to support my claim.	I made some mathematical and/or representational errors that make my argument hard to follow.	I made some mathematical and/or representational errors but my argument is sound.	My evidence is sound and correct.	N/A
Writing Mechanics (x2)	I did not include a written explanation.	My written explanation contains spelling and grammar mistakes that impact the understanding of my argument.	My written explanation contained few spelling and/or grammar mistakes.	My written explanation is complete and uses grade appropriate spelling and grammar.	N/A

	3	2	1
CLAIM  The claim is what is to be shown to be true or not true.  The claim is accurate and clearly stated.		The claim is accurate, but may be unclear or confusing.	The claim is not accurate or not included in the argument.
EVIDENCE The "math". It can take the form of equations, tables, charts, diagrams, graphs, words, symbols, etc.  The evidence supports the claim. It is accurate and complete.		The evidence supports the claim, but may be incomplete or somewhat inaccurate.	The evidence does not support the claim. It is incomplete and/or inaccurate.
WARRANT Explain how the evidence is relevant for the claim. It can be definitions, theorems, agreed upon facts, rules, or properties.	The warrant explains how the evidence supports the claim. It refers to a certain rule that makes the evidence true.	The warrant explains how the evidence supports the claim, but may be incomplete or unclear.	The warrant does not support the evidence, or is not there.
PRECISION  The language used needs to be precise enough to communicate the ideas with sufficient clarity.  The argument is precise. Ma vocabulary is used and the language communicates the ideas clearly.		The argument is somewhat precise. Some math vocabulary is used. The language used communicates the ideas but may be unclear or confusing.	The argument is not precise.  Math vocabulary is not used, and the language is unclear and confusing.
COMPONENTS  Valid mathematical arguments have a claim, evidence, and a warrant.	The mathematical argument has all three components: a claim, evidence, and a warrant.	The mathematical argument has two components.	The mathematical argument has one or zero components.

	My Ma	th Argumentation Rubric		
	Communicating Thinking with  Math Vocabulary	Understanding the Problem	Listening to and Communicating about others' Thinking	
3	I use precise mathematical vocabulary to justify and explain my thinking. My explanation tells what I did and why I did it.	I show I understand the problem by highlighting important terms and phrases and use different math ideas I already know to justify my answer.	I explain other students' thinking and identify their strengths and weaknesses appropriately. I ask my peers questions and listen to their responses.	
2	I describe but do not tell why my response is correct. I use general words instead of specific math vocabulary to explain my thinking.	I put minimal effort in showing I understand the problem and in using different math ideas I know to justify my answer.	I respond to other students' ideas, but it is not clear if I am listening carefully to what they say in order to critique their ideas or ask questions.	
1	I explain or show my thinking for the solution only after being prompted or supported by a teacher or peers.	I do not show that I understand the problem and ask clarifying questions only when prompted by a teacher.	I attempt to listen and understand other students' ideas only when asked by an adult. I ask other students few or no questions about their work.	

Argument Component	Exemplary (3)	Proficient (2)	Developing (1)	Does Not Meet Expectation
Claim	A claim or solution is stated and a clear stance is taken that agrees with the argument.	A claim or solution is stated but a clear stance is not provided.	A claim or solution is implied but not clearly stated in the argument.	A claim or solution is not stated nor implied in the argument.
Evidence	Appropriate, accurate and detailed evidence (mathematical or written) is provided to fully support the claim.	Appropriate and detailed evidence (mathematical or written) is provided to fully support the claim.	Some evidence is provided but it is not detailed enough to fully support the claim.	Limited evidence is provided and it does not clearly or fully support the claim
Warrant	The stance is justified with clear, logical and organized statements to lead the reader through the evidence to support the claim.	The stance is justified with statements that lead the reader from the claim through the evidence. Other statements may be included that do not provide a warrant.	A series of statements are provided which describe and explain the evidence but do not link the evidence to the claim.	Statements provided do not explain the evidence nor link it to the claim.
Accuracy	The evidence provided is sound and mathematically correct.	Some mathematical and/or representational errors may be present but the overall argument is sound.	Some mathematical and/or representational errors occur that make the argument incomplete or difficult to follow.	No evidence is provided to support the claim.
Writing Mechanics	The written explanations in the argument are complete and use grade appropriate spelling and grammar.	The written explanations in the argument contain few spelling and/or grammar mistakes	The written explanations in the argument contain spelling and grammar mistakes that impact the understanding of the argument.	No written explanations are provided in the argument.