# Which Figure Is It? (HS Geometry)

## STUDENT WORK SAMPLE ARGUMENTATION RESOURCE PACKET

This packet was produced as part of the Bridging Math Practices Math-Science Partnership Grant (2014 -2015).

The purpose of the packet is to help a) reveal what students can do with respect to generating an argument in response to mathematical questions, including the variety of their arguments; b) highlight features that should be considered when reviewing students' arguments, and c) identify what counts as a *quality* argument in light of the review criteria.

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## What is a mathematical argument?

A mathematical argument is
a sequence of statements and reasons given with the aim of demonstrating that a claim is true or false.

This links to the Connecticut Core Standards of Mathematical Practice #3, *construct viable arguments and critique the reasoning of others,* as well as other standards.

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This resource packet is a product of work by participants in the UConn Bridging Math Practices Math-Science Partnership Grant, which included faculty and graduate students from the University of Connecticut's Neag School of Education and Department of Mathematics, and teachers and coaches from the Manchester Public Schools, Mansfield Public Schools, and Hartford Public Schools.
This resource packet reflects significant contributions from Jenn Downes, Jeana Favat, Cathy Mazzotta, Belinda Perez, Adrianne Satin, and John Tedesco. Many thanks for all their insights and contributions! For more information about the grant, or for additional argumentation-related materials and resource, please see the project website: http://bridges.uconn.education.edu

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# What is a high quality mathematical argument?

A high quality mathematical argument is an argument that shows that a claim must be true. It leaves little room to question. The chain of logic leads the reader to conclude that the author's claim is true.

What are the characteristics of a high quality argument? A high quality argument can be described by the following components and criteria:

| **Criteria** | **Description** |
|---|---|
| 1. A clearly stated **claim** | The claim is what is to be shown true or not true. |
| 2. The necessary **evidence** to support the claim | Evidence can take the form of equations, tables, charts, diagrams, graphs, words, symbols, etc. It is one's "work" which provides the information to show something is true/false. |
| 3. The necessary **warrants** to connect the evidence to the claim | Warrants can take the form of definitions, theorems, logical inferences, agreed upon facts. Warrants explain how the evidence is relevant for the claim, and collectively they chain the evidence together to show the claim is true or false. |
| 4. **Language use and computations** are at a sufficient level of precision and accuracy | The language used and computations must be at a sufficient level of precision or accuracy to support the argument. Language use needs to be precise enough to communicate the ideas with sufficient clarity. |

These criteria are helpful for discussions. It is important not to lose sight of the "big picture" however, which is whether the argument offered shows that the claim is (or is not) true. This is the goal and purpose of a mathematical argument. You will see in many of these packets that students can approach an argumentation prompt from many different perspectives. It matters less *which* mathematical tools they use, and matters more whether their chain of reasoning compels the result.

In this packet you will find

1. A blank copy of the task (Which Figure Is It?) and a description of the task implementation and/or other important considerations regarding student work samples included in this packet.
2. A protocol that can help you and your colleagues discuss student work related to this task. The use of the protocol is optional.
3. Selected work samples on this task from high school students (grades 9 and 10, across courses) in a school participating in the UConn Bridging Math Practices project to be used with the protocol.
4. Work Samples Classification and Commentaries: the student work samples ordered by whether they seem to be high, adequate, or low quality responses with respect to the criteria described on page 2 along with commentaries that support the classification. Among the samples are some that present a well-structured argument, but have important mathematical flaws, which prevent them from being classified as the highest quality.

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much "weight" people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional discussions and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

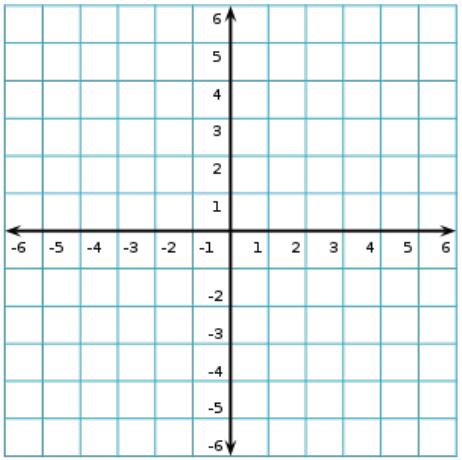
**THE TASK**

The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$.

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

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**CONTEXT**

This problem was given to all 9th- and 10th-grade students at one high school. These students were from a variety of levels and classes ranging from Algebra 1 to Pre-calculus. The purpose of this problem was to see how students combine Algebra and Geometry skills and concepts to justify a solution. We offer one set of commentaries here. Student expectations may vary based on the class and manner in which the task is administered (Algebra 1 vs. Algebra 2 or Geometry). Given the variety of students, implied warrants were at times difficult to determine because there were differences across classes in previously agreed upon background knowledge and established theorems and definitions. It was, however, an assessment situation, and so we expected students to be more explicit than they might be on a class assignment.

The set of 10 student responses are listed from strongest to weakest argument within each of the three categories, high, adequate and low.

# Protocol Guided Sorting Activity: (33–40 mins)
# Bridging Math Practices Math-Science Partnership Grant

This protocol was created for the purpose of reviewing student work. It is modified from two of the previously presented protocols in the Manchester School District. The original protocols apply to when teachers bring their own students' work. This has been modified to review prepared packets of student work.

- Maryland Protocol: Examining Student Work to Inform Instruction – Maryland State Department of Education http://mdk12.org/instruction/examining/protocol.html
- Collaborative Analysis Protocol - San Diego County Board of Education http://plc.sdcoe.net/Resources/Data%20Driven%20Decisions/LASWProtocol_Dec2011Rev.pdf

This is sometimes referred to as a ***Tuning Protocol,*** as the purpose is to help a group align their visions and expectations. Here, the alignment is with respect to the question: what is a high quality argument (on this task, for this grade level)? A main goal of this protocol is to support colleagues in building a consensus around what counts as a high quality argument.

## 0. Assign Roles

**The Handler** – places work samples in agreed-upon pile
**Facilitator** – ensures space is made for all to contribute; supports finding consensus
**Time Keeper** – keeps time and ensures group doesn't exceed section time limits. Can prompt movement to next section even if full time is not used.
**All–** share ideas and keep notes on own set of work samples

## A: Setting the context for discussion (5 mins)

Team members read and do the problem. Team members discuss: What was the "big idea" of the task/assessment? What result or claim needed justification?

## B: Quick sort: Reviewing student work (15 mins)

Do a *Quick Sort* of students' work by the degree of proficiency (high, adequate, low) demonstrated with providing an argument of the relevant claim(s). The Handler places a copy of the student work into the appropriate pile as agreed upon by the group. You may initially need a "Not Sure" pile. After sorting, revisit papers in the "Not Sure" pile and match each with the typical papers in one of the other piles. Record work sample numbers in the appropriate column of the chart (next page).

*The facilitator may also decide to begin the Quick Sort with some silent review of student work samples before starting discussion.*

**Sorting Chart**

| **HIGH Quality** (high quality mathematical argument) | **ADEQUATE Quality** (adequate mathematical argument) | **LOW** (low quality mathematical argument) |
|---|---|---|
| | | |

## C: Strengths and areas for growth? (5 mins)

Group member summarize key ideas from their Sorting Discussion regarding the strengths and areas for growth for individual samples, each group[1] (High Quality, Adequate, Low) of samples, or the overall set with respect to the argumentation?

| **HIGH Quality** (high quality mathematical argument) | **ADEQUATE Quality** (adequate mathematical argument) | **LOW** (low quality mathematical argument) |
|---|---|---|
| | | |
| **Strengths overall for the class** | | |

[1] This question is phrased in terms of "subgroups." You may or may not be able to characterize the group as a whole. As needed, describe individual or pairs of student work.

## D: Reading ARP Commentaries (optional: 5-7 mins)

As deemed useful, group members read the commentaries in the Argumentation Resource Packet to gain new perspectives on selected student work samples, their strengths and areas for growth, and what counts as a high quality argument.

## E: Reflection (5 mins) *Each person shares*

The facilitator guides the group to take turns in sharing a reflection. Group may decide to reflect on the same question, or each share a take away.

a. What did you learn about argumentation and how students engage argumentation from looking at the work of these students? You might also consider aspects of task design.

b. Did you have any *ah hah* moments?

c. What questions remain for you? What would you like to lean more about?

d. What will you take away from this discussion back to your classroom? What ideas might impact your planning or teaching?

## F: Reflection on Protocol Implementation (3 mins)

Facilitator guides a reflection on how the protocol process worked. Group members contribute ideas. Members make suggestions for modifications to future protocol as needed.

# Student 1

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

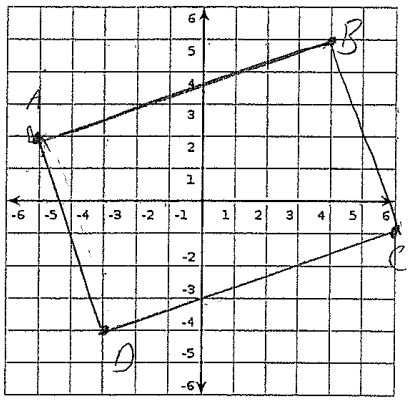
**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

Slope A-D (-3)
Slope B-C -3
Slope AB 1/3
Slope C-D 1/3

Length A-D $2\sqrt{10}$
length B-C $2\sqrt{10}$
Length A-B $2\sqrt{10}$
length C-D $2\sqrt{10}$

$\sqrt{(3-5)^2+(4-2)^2}$
$\sqrt{4+36}$
$\sqrt{(6-4)^2+(-1-5)^2}$
$\sqrt{4+36}$




The Shape is a square. Through mathematical calculations, all side lengths are $2\sqrt{10}$ and the slope of each line is its reciprical. Slope of A-D is -3 and Slope of C-D is 1/3. The side lengths are all equal and angles 90° due to slope. The characteristics of a square define this.

# Student 2

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$\sqrt{(4-(-5))^2 + (5-2)^2}$

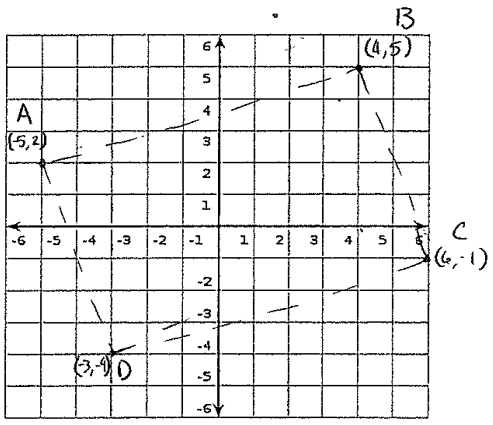
$9^2 + 3^2$

$81 + 9$

$\sqrt{90}$

$\overline{AB} \cong \overline{DC}$

$\overline{AD} \cong \overline{BC}$




$\sqrt{(6-(-3))^2 + (-1-(-4))^2}$

$9^2 + 3^2$

$81 + 9$

$\sqrt{90}$

$m = \frac{5-2}{4-(-5)} = \frac{3}{9} = \frac{1}{3}$ slope $AB \cong$ slope $DC$

$m = \frac{-1-(-4)}{6-(-3)} = \frac{3}{9} = \frac{1}{3}$

$\sqrt{(-3-(-5))^2 + (-4-2)^2}$

$2^2 + (-6^2)$

$4 + 36$

$\sqrt{40}$

$m = \frac{2-(-4)}{-5-(-3)} = \frac{6}{-2}$ slope $AD \cong$ slope $BC$

$m = \frac{5-(-1)}{4-6} = \frac{6}{-2}$

$\sqrt{(6-4)^2 + (-1-5)^2}$

$2^2 + (-6^2)$

$4 + 36$

$\sqrt{40}$

The shape is a rectangle

# Student 3

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$\overline{AB}$: $m = \frac{5-2}{4+5} = \frac{3}{9} = \frac{1}{3}$

$d = \sqrt{(4+5)^2 + (5-2)^2}$

$d = \sqrt{81+9}$

$d = \sqrt{90}$

$d = 3\sqrt{10}$

$\overline{DC}$: $m = \frac{-1+4}{6+3} = \frac{3}{9} = \frac{1}{3}$

$d = \sqrt{(9)^2 + (3)^2}$

$d = \sqrt{90}$

$d = 3\sqrt{10}$

$\overline{BC}$: $m = \frac{5+1}{4-6} = \frac{6}{2} = -3$

$d = \sqrt{(-2)^2 + (6)^2}$

$d = 2\sqrt{10}$

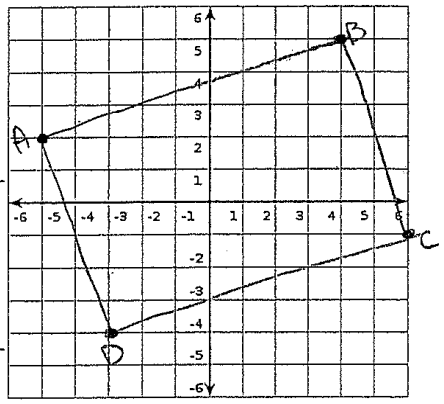
$\overline{AD}$: $m = \frac{2+4}{-5+3} = -\frac{6}{2} = -3$

$d = \sqrt{(-2)^2 + (6)^2}$

$d = \sqrt{4+36}$

$d = \sqrt{40}$

$d = 2\sqrt{10}$




This quadrilateral is a rectangle

By definition, a rectangle is a quadrilateral in which there are 2 sets of sides that are the same length and parallel and 4 right angles. There are 2 sets of sides that are parallel + the same length. $\overline{AB}$ is parallel to $\overline{CD}$ with the slope $\frac{1}{3}$, and both are the same length ($3\sqrt{10}$). $\overline{AD}$ is parallel to $\overline{BC}$ with the slope $-3$, and both of these lines are also the same length ($2\sqrt{10}$). I also know that this figure has all right angles because $\overline{AD}$ and $\overline{BC}$, with slopes of $-3$, are perpendicular to lines $\overline{AB}$ and $\overline{CD}$, with slopes of $\frac{1}{3}$. When lines are perpendicular, they create right angles. Based on all of this information, this quadrilateral can be proven to be a rectangle.

# Student 4

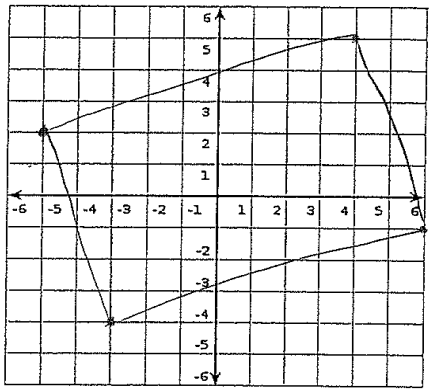
2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

Obviously, just by graphing the vertices you can see that this quadrilateral is a rectangle.



It has two sets of parallel lines, four 90° angles, yet 2 sides lengths are longer than the other set.

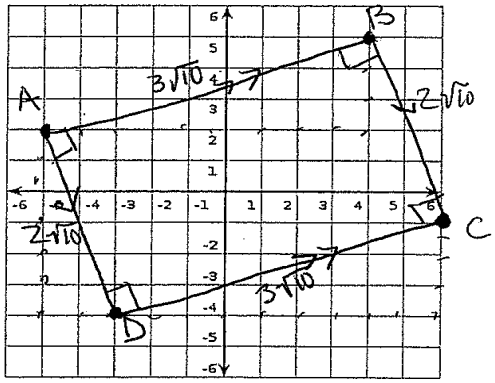
# Student 5

) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

se slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write nathematical argument to justify your classification.




slope $\overline{AB} = \frac{3}{9} = \frac{1}{3}$

slope $\overline{DC} = \frac{3}{9} = \frac{1}{3}$ $\therefore \overline{AB} \parallel \overline{DC}$

slope $\overline{AD} = \frac{-6}{2} = -3$

slope $\overline{BC} = \frac{-6}{2} = -3$ $\therefore \overline{AD} \parallel \overline{BC}$

$\therefore \angle A = 90^\circ$ $\overline{AB} \parallel \overline{DC}$

$\angle B = 90^\circ$ $\overline{AD} \parallel \overline{BC}$

$\angle C = 90^\circ$ $\overline{AB} \approx \overline{DC}$

$\angle D = 90^\circ$ $\overline{AD} \approx \overline{BC}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\overline{AB} = \sqrt{(4+5)^2 + (5-2)^2}$

$\overline{AB} = \sqrt{9^2 + 3^2}$

$\overline{AB} = \sqrt{81+9}$

$\overline{AB} = \sqrt{90}$

$\overline{AB} = 3\sqrt{10}$ $\therefore \overline{DC} = 3\sqrt{10}$

Rectangle:

using the slope formula, I found that $\overline{AB}$ and $\overline{DC}$ are parallell because the have the same slope. $\overline{AD} \parallel \overline{BC}$ as well because they have the same slope. The slopes are opposite and flipped over so all angles are 90°. However, since $\overline{BC} \not\approx \overline{AB}$ and $\overline{AD} \not\approx \overline{DC}$ all sides are not equal making it a rectangle

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

~~$\overline{BC} = \sqrt{(6+1)^2 + (5-6)}$~~

$\overline{BC} = \sqrt{(6-9)^2 + (-1-5)^2}$

$\overline{BC} = \sqrt{2^2 + (-6)^2}$

$\overline{BC} = \sqrt{4+36}$

$\overline{BC} = \sqrt{40}$

$\overline{BC} = 2\sqrt{10}$ $\therefore \overline{AD} = 2\sqrt{10}$

# Student 6

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

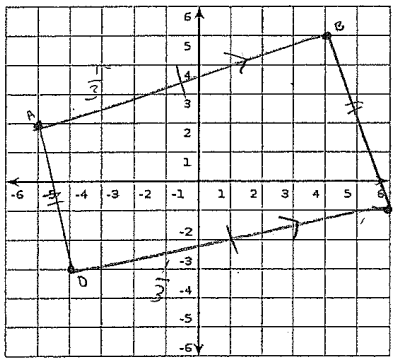
Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$\frac{5-2}{4+5}$ $\frac{3}{9}$ $\frac{1}{3}$

$\frac{-4+1}{-3-6}$ $\frac{-3}{-9}$ $\frac{1}{3}$




$\frac{-4-2}{-3+5}$ $\frac{-6}{2}$ $-3$

$\frac{-1-5}{6-4}$ $\frac{-6}{2}$ $-3$

the shape is a parallelogram because its opposite sides are both congruent & parallel. I found this using distance formula to find the length of the segment & the slope formula to see if they were parallel.

$\sqrt{(-5+4)^2 + (2-5)^2}$

$\sqrt{(-9)^2 + (-3)^2}$

$\sqrt{81+9}$

$\sqrt{90}$

$9.49$

$\sqrt{(-5+3)^2 + (2+4)^2}$

$\sqrt{(-2)^2 + (6)^2}$

$\sqrt{4+36}$

$\sqrt{40}$

$6.32$

$\sqrt{(4-6)^2 + (5+1)^2}$

$\sqrt{(-2)^2 + (6)^2}$

$\sqrt{4+36}$

$\sqrt{40}$

$6.32$

$\sqrt{(9)^2 + (3)^2}$

$\sqrt{81+9}$

$\sqrt{90}$

$9.49$

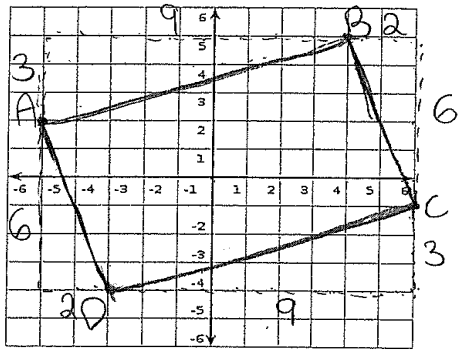
# Student 7

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.




$\frac{-1-5}{6-4} = \frac{-6}{2} = -3$

$\overline{AD}$ and $\overline{BC}$ have the same slope

$\frac{5-2}{4++5} \quad \frac{3}{9} = 1/3$

$\frac{-4+1}{-3-6} \quad \frac{-3}{-9} = 1/3$

$\overline{AB}$ and $\overline{DC}$ have the same slope

$9^2 + 3^2 = C^2$ 7811

$9^2 + 3^2 = C^2$

$6^2 + 2^2 = C^2$ 1600

$6^2 + 2^2 = C^2$

Using the pathagorem therom you can tell both are the same.

# Student 8

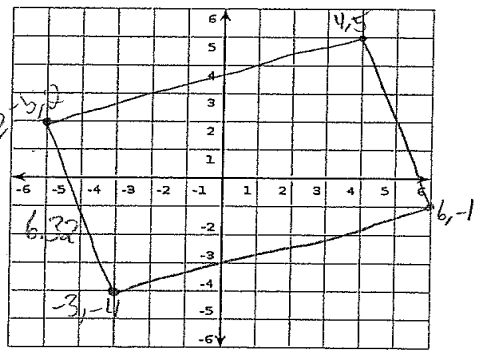
2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

This shape is a rectangle, It has equal opposite sides, and congruent opposite slopes.




$d = \sqrt{(-4-2)^2 + (-3-5)^2}$

$d = \sqrt{(-6)^2 + (2)^2} = \sqrt{36+4} = \sqrt{40}$

$= 6.32$

$d = \sqrt{(4-6)^2 + (5--1)^2}$

$d = \sqrt{(-2)^2 + (-6)^2} = \sqrt{36+4} = \sqrt{40}$

$= 6.32$

$\frac{5-2}{4--5} = \frac{3}{9} = \frac{1}{3} = \text{slope}$

$\frac{-1--4}{6--3} = \frac{3}{9} = \frac{1}{3} = \text{slope}$ } = length

$\frac{-4-2}{-3--5} = \frac{-6}{2} = \frac{-3}{1} = \text{slope}$

$\frac{-1-5}{6-4} = \frac{-6}{2} = \frac{-3}{1} = \text{slope}$ } = width

# Student 9

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$(-3,-4)(6,-1)$

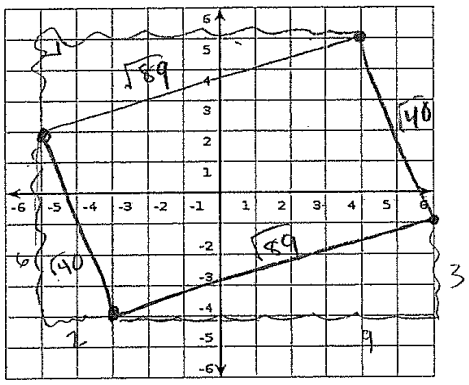
$m = \frac{6 - -3}{-1 - -4} \quad \frac{9}{3} = 3$

$m = (-5,2)(-4,-3)$

$\frac{-4 - -5}{-3 - 2} = \frac{1}{-5}$

$m = (4,5)(6,-1)$

$\frac{6-5}{-1-4} = \frac{1}{-5}$




$4 + 36 = \sqrt{40}$ $\quad 9 + 81 = \sqrt{89}$

This quadrilateral is a rectangle because there are 2 sets of congruent sides. The length is $\sqrt{89}$ and the width is $\sqrt{40}$. The slope of the length is 3 and the widths $-\frac{1}{5}$. The 2 sets of congruent sides (Length and width size difference) eliminates rhombus and square since all sides are equal for them. Trapezoid is eliminated because it has 2 acute and 2 obtuse angles which this quad. lacks. Between rectangle & parallelogram it is rectangle because it appears to lack obtuse and acute angles.

# Student 10

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

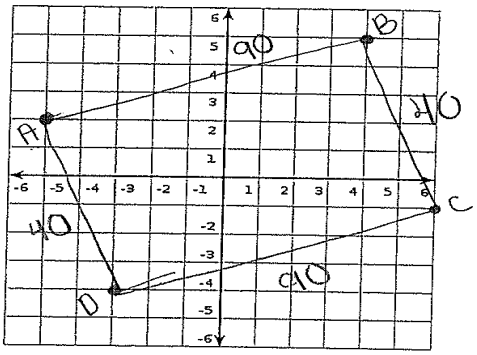
$\frac{-4+2}{-3+5} = \frac{-6}{2} = -3$

$\frac{5+2}{4+5} = \frac{3}{9} = 3$

$\frac{-1+5}{6+4} = \frac{-6}{2} = -3$

$\frac{-1+-4}{6+3} = \frac{3}{9} = 3$

AD = -3
AB = 3
BC = -3
DC = 3




$-3+5 = 2^2$ $\quad -4+2 = -6^2$

$4+36 = 40$ AD

$6+4 = 2^2$ $\quad -1+5 = -6^2$

$4+36 = 40$ BC

$4+5 = 9^2$ $\quad 5+2 = 3^2$

$81+9 = 90$ AB

$-3+6 = -9^2$ $\quad -4+1 = -3^2$

$81+9 = 90$ DC

# Which Figure Is It? (Geometry)

ANNOTATED

# STUDENT WORK SAMPLE ARGUMENTATION RESOURCE PACKET

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much "weight" people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional discussions and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

# Student 3

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$\overline{AB}$: $m = \frac{5-2}{4+5} = \frac{3}{9} = \frac{1}{3}$

$d = \sqrt{(4+5)^2 + (5-2)^2}$

$d = \sqrt{81+9}$

$d = \sqrt{90}$

$d = 3\sqrt{10}$

$\overline{DC}$: $m = \frac{-1+4}{6+3} = \frac{3}{9} = \frac{1}{3}$

$d = \sqrt{(9)^2 + (3)^2}$

$d = \sqrt{90}$

$d = 3\sqrt{10}$

$\overline{BC}$: $m = \frac{5+1}{4-6} = \frac{6}{-2} = -3$

$d = \sqrt{(-2)^2 + (6)^2}$

$d = 2\sqrt{10}$

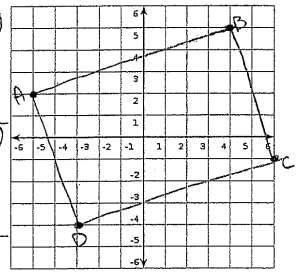
$\overline{AD}$: $m = \frac{2+4}{-5+3} = -\frac{6}{2} = -3$

$d = \sqrt{(-2)^2 + (6)^2}$

$d = \sqrt{4+36}$

$d = \sqrt{40}$

$d = 2\sqrt{10}$




This quadrilateral is a rectangle

By definition, a rectangle is a quadrilateral in which there are 2 sets of sides that are the same length and parallel and 4 right angles. There are 2 sets of sides that are parallel + the same length. $\overline{AB}$ is parallel to $\overline{CD}$ with the slope $\frac{1}{3}$, and both are the same length ($3\sqrt{10}$). $\overline{AD}$ is parallel to $\overline{BC}$ with the slope $-3$, and both of these lines are also the same length ($2\sqrt{10}$). I also know that this figure has all right angles because $\overline{AD}$ and $\overline{BC}$, with slopes of $-3$, are perpendicular to lines $\overline{AB}$ and $\overline{CD}$, with slopes of $\frac{1}{3}$. When lines are perpendicular, they create right angles. Based on all of this information, this quadrilateral can be proven to be a rectangle.

## Commentary

This student's argument was categorized as **HIGH quality.**

This student makes the claim of Rectangle. The student's argument rests on a definition of rectangle (the warrant), which the student makes explicit. The student correctly calculates slopes and side lengths, and also interprets the information correctly to show that the figure has the needed features (opposite, parallel sides and 4 right angles).

The response could be strengthened by the student more clearly identifying that opposite sides are congruent, but consecutive sides not congruent. The response could also be strengthened by explicitly stating that perpendicular line segments result when segments have slopes with values that are negative reciprocals.

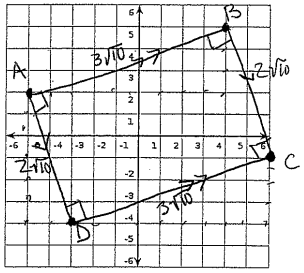
# Student 5

) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

se slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write nathematical argument to justify your classification.




slope $\overline{AB} = \frac{3}{9} = \frac{1}{3}$ ∴ $\overline{AB} \parallel \overline{DC}$

slope $\overline{DC} = \frac{3}{9} = \frac{1}{3}$

slope $\overline{AD} = \frac{-6}{2} = -3$ ∴ $\overline{AD} \parallel \overline{BC}$

slope $\overline{BC} = \frac{-6}{2} = -3$

∴ $\angle A = 90°$ $\overline{AB} \parallel \overline{DC}$
$\angle B = 90°$ $\overline{AD} \parallel \overline{BC}$
$\angle C = 90°$ $\overline{AB} \approx \overline{DC}$
$\angle D = 90°$ $\overline{AD} \propto \overline{BC}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\overline{AB} = \sqrt{(4+5)^2 + (5-2)^2}$

$\overline{AB} = \sqrt{9^2 + 3^2}$

$\overline{AB} = \sqrt{81 + 9}$

$\overline{AB} = \sqrt{90}$

$\overline{AB} = 3\sqrt{10}$ ∴ $\overline{DC} = 3\sqrt{10}$

Rectangle:
using the slope formula, I found that $\overline{AB}$ and $\overline{DC}$ are parallel because the have the same slope: $\overline{AD} \parallel \overline{BC}$ as well because they have the same slope. The slopes are opposite and flipped over so all angles are 90°. However, since $\overline{BC} \not\approx \overline{AB}$ and $\overline{AD} \not\approx \overline{DC}$ all sides are not equal making it a rectangle

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\overline{BC} = \sqrt{(6-4)^2 + (-1-5)^2}$

$\overline{BC} = \sqrt{2^2 + (6)^2}$

$\overline{BC} = \sqrt{4 + 36}$

$\overline{BC} = \sqrt{40}$

$\overline{BC} = 2\sqrt{10}$ ∴ $\overline{AD} = 2\sqrt{10}$



## Commentary

This student's argument was categorized as **HIGH quality**.

This student makes the claim of Rectangle and has evidence to support this: the student properly calculates slopes, infers angle measures from the slopes, and properly calculates side lengths, as well as interprets those calculations correctly. The student also identifies that not all sides are congruent, ruling out square.

Though the student doesn't state a definition for a rectangle (warrant), the student implies that if the following three criteria are met, then it is a rectangle: (1) opposite sides are parallel, (2) the angles are 90 degrees, and (3) if 2 pairs of sides have congruent length and not all 4 sides are congruent in length. [Note that (1) and (2) are needed to show the quadrilateral is a rectangle, and (3) shows that it is a rectangle that is also not a square.]

The response could be strengthened by the student more clearly identifying that not only are consecutive sides not congruent, but opposite sides *are* congruent, which is not explicitly stated, but implied by the comment that not all sides were congruent. The response could also be strengthened by a clear articulation of a definition of a rectangle.

# Student 6

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

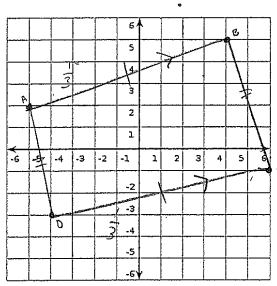
Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$\frac{5-2}{4+5}$ $\frac{3}{9}$ $\frac{1}{3}$

$\frac{-4+1}{-3-6}$ $\frac{-3}{-9}$ $\frac{1}{3}$




$\frac{-4-2}{-3+5}$ $\frac{-6}{2}$ $-3$

$\frac{-1-5}{6-4}$ $\frac{-6}{2}$ $-3$

the shape is a parallelogram because its opposite sides are $\sqrt{(-5+4)^2 + (2-5)^2}$ both congruent & parallel. I found this using distance formula to find the length $\sqrt{(-9)^2 + (-3)^2}$ of the segment & the slope formula to see $\sqrt{81 + 9}$ if they were $\sqrt{(6+3)^2 + (-1+4)^2}$ Parallel.

$\sqrt{(-5+3)^2 + (2+4)^2}$ $\sqrt{(9-6)^2 + (5+1)^2}$ $\sqrt{(9)^2 + (3)^2}$ $\sqrt{90}$ 9.49

$\sqrt{(-2)^2 + (6)^2}$ $\sqrt{(-2)^2 + (6)^2}$ $\sqrt{81 + 9}$

$\sqrt{4 + 36}$ $\sqrt{4 + 36}$ $\sqrt{90}$

$\sqrt{40}$ $\sqrt{40}$ 9.49

6.32 ≅ 6.32

## Commentary

This student's argument was categorized as **ADEQUATE quality**.

This student makes the claim of Parallelogram and has evidence and warrant to support this claim. The student properly calculates slope and side lengths, and properly interprets this information to mean opposite sides are congruent and parallel. The student also offers a correct definition of parallelogram and shows that the necessary criteria are met. The argument that this is a *parallelogram* is complete. We did not factor in the wrong plot of point D since this was not used in the argument, and did not diminish the strength of the argument.

As the claim is for *the most specific classification possible,* the response could be extended by the student making explicit why the figure is not also a rhombus or any other shape listed. Furthermore, the response could be extended by recognizing the slope values indicate that adjacent sides are perpendicular creating 90 degree angles, ruling out parallelogram as the ***most specific*** classification of the quadrilateral.

# Student 8

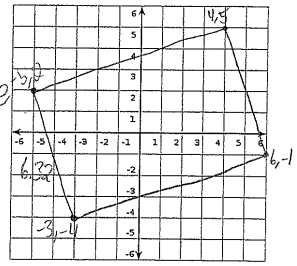
2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

This shape is a rectangle, It has equal opposite sides, and congruent opposite slopes.




$d = \sqrt{(-4-2)^2 + (-3-5)^2}$

$d = \sqrt{(-6)^2 + (2)^2} = \sqrt{36+4} = \sqrt{40}$

$= 6.32$

$d = \sqrt{(4-6)^2 + (5--1)^2}$

$d = \sqrt{(-2)^2 + (-6)^2} = \sqrt{36+4} = \sqrt{40}$

$= 6.32$

$\frac{5-2}{4--5} = \frac{3}{9} = \frac{1}{3} = \text{slope}$

$\frac{-1--4}{6--3} = \frac{3}{9} = \frac{1}{3} = \text{slope}$ } = length

$\frac{-4-2}{-3--5} = \frac{-6}{2} = \frac{-3}{1} = \text{slope}$

$\frac{-1-5}{6-4} = \frac{-6}{2} = \frac{-3}{1} = \text{slope}$ } = width

## Commentary

This student's argument was categorized as **ADEQUATE quality**.

This student makes the claim of Rectangle and has evidence and an (implied) warrant to support this. (The implied warrant is a definition or theorem about rectangles: a rectangle is a figure with (a) equal opposite sides and (b) slopes that are negative reciprocal. Though accurate, it requires more information than needed: showing the quadrilateral has four right angles is sufficient to conclude it is a rectangle.) The student argues the figure is a rectangle, but does not argue that a rectangle is the *most specific* figure.

Note that the phrase "congruent opposite slopes" can be interpreted in different ways. One interpretation of "congruent opposite slopes" is that the two slopes have "opposite" values (negative reciprocals) which implies the figure is a rectangle (as interpreted above); the other interpretation is that the opposite sides have the same slope (which implies a parallelogram).

The response could be strengthened by the use of a more precise language in the warrant (e.g., revising "congruent opposite slopes"). The student could also make more clear how the second pair of opposite sides are known to be congruent, as these calculations are not shown. (To do this, the student could do the calculations or reference that having one pair of congruent sides and parallel slopes implies the other pair of opposite sides must be congruent.)

Finally, the response could be extended by providing evidence that the figure is a rectangle *and not a square,* which is needed to justify the rectangle as the *most specific* classification.

# Student 1

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

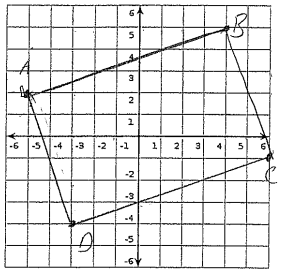
**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

Slope A-D (-3)
Slope B-C -3
Slope A-B 1/3
Slope C-D 1/3

Length A-D $2\sqrt{10}$
Length B-C $2\sqrt{10}$
Length A-B $2\sqrt{10}$
Length C-D $2\sqrt{10}$

$\sqrt{(3--3)^2+(4-2)^2}$
$\sqrt{4+36}$
$\sqrt{(6-4)^2+(1-3)^2}$
$\sqrt{4+36}$




The Shape is a square. Through mathematical calculations, all side lengths are $2\sqrt{10}$ and the slope of each line is its reciprical. Slope of A-D is -3 and Slope of C-D is 1/3. The side lengths are all equal and angles 90° due to slope. The characteristics of a square define this.

## Commentary

This student's argument was categorized as **ADEQUATE quality**.

This student makes the claim of Square and has some evidence and warrant (a definition of a square) to support this. The student's definition of square is: side lengths are all equal and angles are 90 degrees. The student properly identifies the slopes, though no calculations are shown, and what the slopes mean in terms of angle measures. The student calculates the side length for one pair of opposite sides, and then asserts the other pair is also the same length. If that were true, the figure indeed would be a square. This assumed piece of evidence, however, leads to the incorrect claim.

The response could be strengthened by the student labeling their side length calculations. If all calculations were labeled properly, the student might have noticed they did not have enough side lengths to support their claim, thus potentially changing their claim.

The response could also be strengthened by including some indication of how the slope values were derived either from the coordinates provided or the diagram. Whether this needs to be included as part of the evidence, however, many depend on the agreed-upon requirements of an argument for a class.

# Student 9

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

$(-3,-4)(6,-1)$

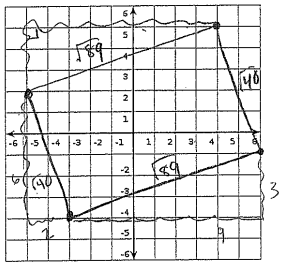
$m = \frac{6 - -3}{-1 - -4} \quad \frac{9}{3} = 3$

$m = (-5,2)(-4,-3)$

$\frac{-4 - -5}{-3 - 2} = \frac{1}{-5}$

$m = (4,5)(6,-1)$

$\frac{6-5}{-1-4} = \frac{1}{-5}$




$4+7 = \sqrt{40}$ $\quad 9+8 = \sqrt{89}$

This quadrilateral is a rectangle because there are 2 sets of congruent sides. The length is $\sqrt{89}$ and the width is $\sqrt{40}$. The slope of the length is 3 and the width $-\frac{1}{5}$. The 2 sets of congruent sides (Length and width size difference) eliminates rhombus and square since all sides are equal for them. Trapezoid is eliminated because it has 2 acute and 2 obtuse angles which this quad. lacks. Between rectangle & parallelogram it is rectangle because it appears to lack obtuse and acute angles.

May 2015

## Commentary

This student's argument was categorized as **borderline of ADEQUATE and LOW quality**.

This student makes the claim of Rectangle and uses an incomplete definition of a rectangle (warrant) to support this claim. The student says a rectangle is a quadrilateral if "there are 2 sets of congruent sides." The student has evidence to support this, showing sqrt 40 and sqrt 89 as the side lengths. The student also appropriately eliminates some of the other classifications, square and rhombus, "since all sides are equal for them." The student, however, makes assumptions about angle measurements of this quadrilateral without proper evidence to eliminate other figures (e.g., trapezoid and parallelogram). There are also calculation errors in the evidence (both in slope and side length).

The response could be strengthened by the student appropriately interpreting the slope calculations. The student calculates the values, but does not indicate what these values mean. (The student asserts the figure "appears to lack obtuse and acute angles." This is true, but not implied by the slope calculations.) If the student had properly interpreted the slope, then the miscalculation may have been caught or would have led to a different claim.

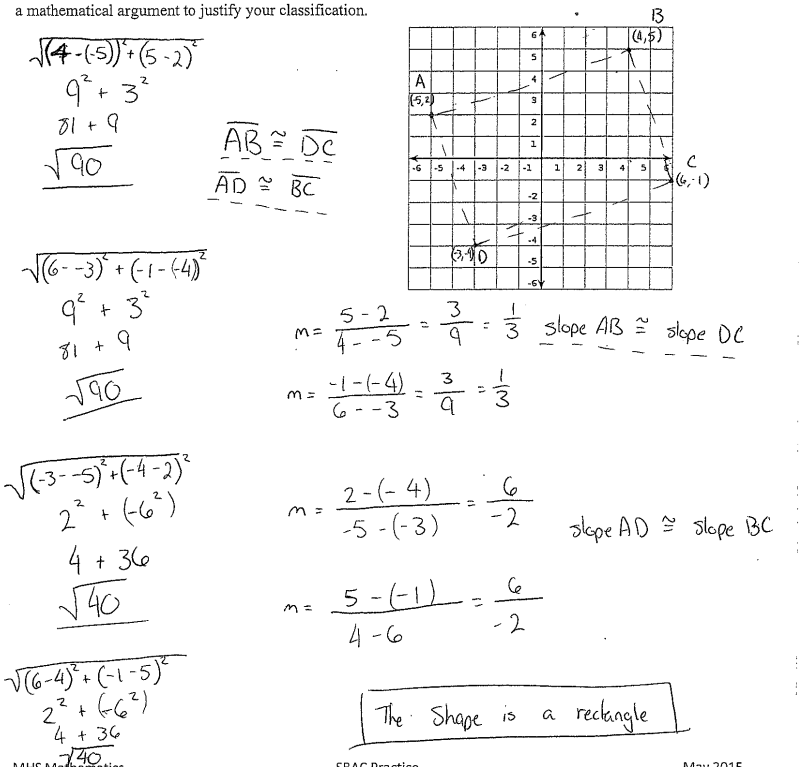
# Student 2

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.




## Commentary

This student's argument is classified as **LOW quality**.

This student makes the claim of Rectangle and has evidence to potentially support this claim. However, the lack of a clear warrant in this argument leads to a classification of low. It appears that some conclusions have been formed based on the evidence (e.g., congruence of pairs of sides) though there is no tie between the conclusions made and definition of a rectangle.

The response could be strengthened by the student providing a warrant, most likely in the form of a definition of a rectangle, to connect the evidence to the claim. The student has accurate calculations to show that this figure has (a) congruent opposite sides, with each pair being different lengths, and (b) sides that meet at right angles, as adjacent sides have slopes values that are negative reciprocals. Together (a) and (b) satisfy the definition of a rectangle, and so articulating this definition and the connection between the evidence and these two criteria would be sufficient to demonstrate the figure is a rectangle.

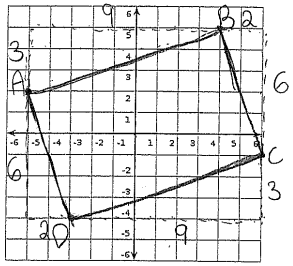
# Student 7

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.




$\frac{-1-5}{6-4} = \frac{-6}{2} = -3$

$\overline{AD}$ and $\overline{BC}$ have the same slope

$\frac{5-2}{4+5} \quad \frac{3}{9} = \frac{1}{3}$

$\frac{-4+1}{-3-6} \quad \frac{-3}{-9} = \frac{1}{3}$

$\overline{AB}$ and $\overline{DC}$ have the same slope

$9^2+3^2=C^2$ 7811

$9^2+3^2=C^2$

$6^2+2^2=C^2$ 7 1600

$6^2+2^2=C^2$

Using the pathagorem theorm you can tell both are the same.

## Commentary

This student's argument was categorized as **LOW quality**.

This student makes no claim in this argument (no selection of a shape). The student does, however, show evidence (using the Pythagorean Theorem) to establish that lengths of opposite sides are the same (without fully calculating the lengths) and uses the slope formula to find the slopes of the sides, and identify slopes that are the same. The evidence, however, is not interpreted (e.g., AB and DC have the same slope, but the student doesn't say they are parallel) or used to suggest a classification for a figure.

The response could be strengthened by the student first making a claim and then providing a warrant to connect the evidence to that claim. There is a mention of opposite sides having the same slope and the same length. A warrant would connect these pieces of evidence to the definition of a rectangle.

# Student 4

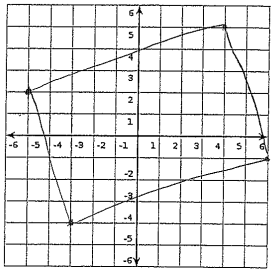
2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

Obviously, just by graphing the vertices you can see that this quadrilateral is a rectangle.



It has two sets of parallel lines, four 90° angles, yet 2 sides lengths are longer than the other set.

## Commentary

This student's argument was categorized as **LOW quality**.

This student makes the claim of Rectangle and supports that claim with a warrant that shows knowledge of the properties of a rectangle and what distinguishes a rectangle from a square. However, there is no evidence that shows the criteria of the warrant is met, and therefore the claim is unsubstantiated.

The response could be strengthened by the student providing work (evidence) to show the two sets of parallel line segments, 90° angles and difference in sides lengths - all referenced by the student.

The response could also be strengthened by a more concise definition of a rectangle (rather than a list of its properties) as one needs only to show four 90o angles to demonstrate the quadrilateral is a rectangle, and then unequal opposite sides to rule out square as the most specific classification.

# Student 10

2.) The vertices of quadrilateral $ABCD$ are $A(-5,2), B(4,5), C(6,-1), D(-3,-4)$

Given the coordinates of the vertices of a quadrilateral, classify the quadrilateral as one of the following using the most specific classification possible:

**Parallelogram, Rectangle, Rhombus, Square, Trapezoid**

Use slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and segment length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to help make your selection then write a mathematical argument to justify your classification.

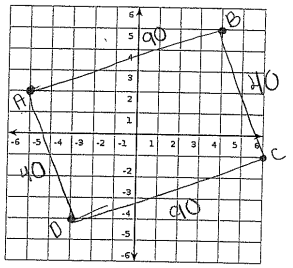
$\frac{-4+2}{-3+5} = \frac{-6}{2} = -3$

$\frac{5+2}{4+5} = \frac{3}{9} = 3$

$\frac{-1+5}{6+4} = \frac{-6}{2} = -3$

$\frac{-1+-4}{6+3} = \frac{3}{9} = 3$

AD = -3
AB = 3
BC = -3
DC = 3




$-3+5 = 2^2$ $-4-2 = -6^2$
$4+36 = 40$ AD
$6+4 = 2^2$ $-1+5 = -6^2$
$4+36 = 40$ BC

$4+5 = 9^2$ $5-2 = 3^2$
$81+9 = 90$ AB
$-3+6 = -9^2$ $-4+1 = -3^2$
$81+9 = 90$ DC

## Commentary

This student's argument was categorized as **LOW quality**.

This student makes the claim of Rectangle and has some evidence to support this. However, the lack of a warrant in this argument leads to a classification of low. There is no explanation that connects the evidence to the claim of rectangle. There are mathematical errors: there is a slight mathematical error in the slope (3/9 = 3) and the student did not take the square root in the calculation of side lengths.

The response could be strengthened by articulating a definition of a rectangle (or its properties) and interpreting the meaning of the calculations in relation to slopes and lengths of sides to show that the figure is a rectangle. For example, the side lengths were calculated but not interpreted in relation to opposite sides being congruent. Similarly there is no mention of parallel sides nor right angles. Finally, the response could be strengthened by addressing how one knows rectangle is *the most specific* classification of the figure.

# Key Connecting Sorting Packet to Argumentation Resource Packet

| Student number (Sorting Packet) | Resource Packet Sample (Quality) |
|---|---|
| 1 | Adequate |
| 2 | Low |
| 3 | High |
| 4 | Low |
| 5 | High |
| 6 | Adequate |
| 7 | Low |
| 8 | Adequate |
| 9 | Border A/L |
| 10 | Low |

| Resource Packet Samples (Quality) | Student number (Sorting Packet) |
|---|---|
| High | 3 |
| High | 5 |
| Adequate | 6 |
| Adequate | 8 |
| Adequate | 1 |
| Adequate/Low | 9 |
| Low | 2 |
| Low | 7 |
| Low | 4 |
| Low | 10 |