Laura Says ¼th is Shaded Problem (Grade 4)
STUDENT WORK SAMPLE ARGUMENTATION RESOURCE PACKET

This packet was produced as part of the Bridging Math Practices Math-Science Partnership Grant (2014 -2015).

The purpose of the packet is to help a) reveal what students can do with respect to generating an argument in response to mathematical questions, including the variety of their arguments; b) highlight features that should be considered when reviewing students' arguments, and c) identify what counts as a *quality* argument in light of the review criteria.

What is a mathematical argument?

A mathematical argument is
a sequence of statements and reasons given with the aim of demonstrating that a claim is true or false.

This links to the Connecticut Core Standards of Mathematical Practice #3, *construct viable arguments and critique the reasoning of others,* as well as other standards.

This resource packet is a product of work by participants in the UConn Bridging Math Practices Math-Science Partnership Grant, which included faculty and graduate students from the University of Connecticut's Neag School of Education and Department of Mathematics, and teachers and coaches from the Manchester Public Schools, Mansfield Public Schools, and Hartford Public Schools. This resource packet reflects significant contributions from An'drea Flynn, Christine Giaquinto, Kylie Hoke, Teresa Maturino, and Diane Ozmun. Many thanks for all their insights and contributions! For more information about the grant, or for additional argumentation-related materials and resource, please see the project website: http://bridges.uconn.education.edu
The Mathematics and Science Partnership (MSP) grant is a federal program funded under Title II, Part B, of the *Elementary and Secondary Education Act* and administered by the U.S. Department of Education (ED).

What is a high quality mathematical argument?

A high quality mathematical argument is an argument that shows that a claim must be true. It leaves little room to question. The chain of logic leads the reader to conclude that the author's claim is true.

What are the characteristics of a high quality argument? A high quality argument can be described by the following components and criteria:

Criteria	**Description**
1. A clearly stated **claim**	The claim is what is to be shown true or not true.
2. The necessary **evidence** to support the claim	Evidence can take the form of equations, tables, charts, diagrams, graphs, words, symbols, etc. It is one's "work" which provides the information to show something is true/false.
3. The necessary **warrants** to connect the evidence to the claim	Warrants can take the form of definitions, theorems, logical inferences, agreed upon facts. Warrants explain how the evidence is relevant for the claim, and collectively they chain the evidence together to show the claim is true or false.
4. **Language use and computations** are at a sufficient level of precision and accuracy	The language used and computations must be at a sufficient level of precision or accuracy to support the argument. Language use needs to be precise enough to communicate the ideas with sufficient clarity.

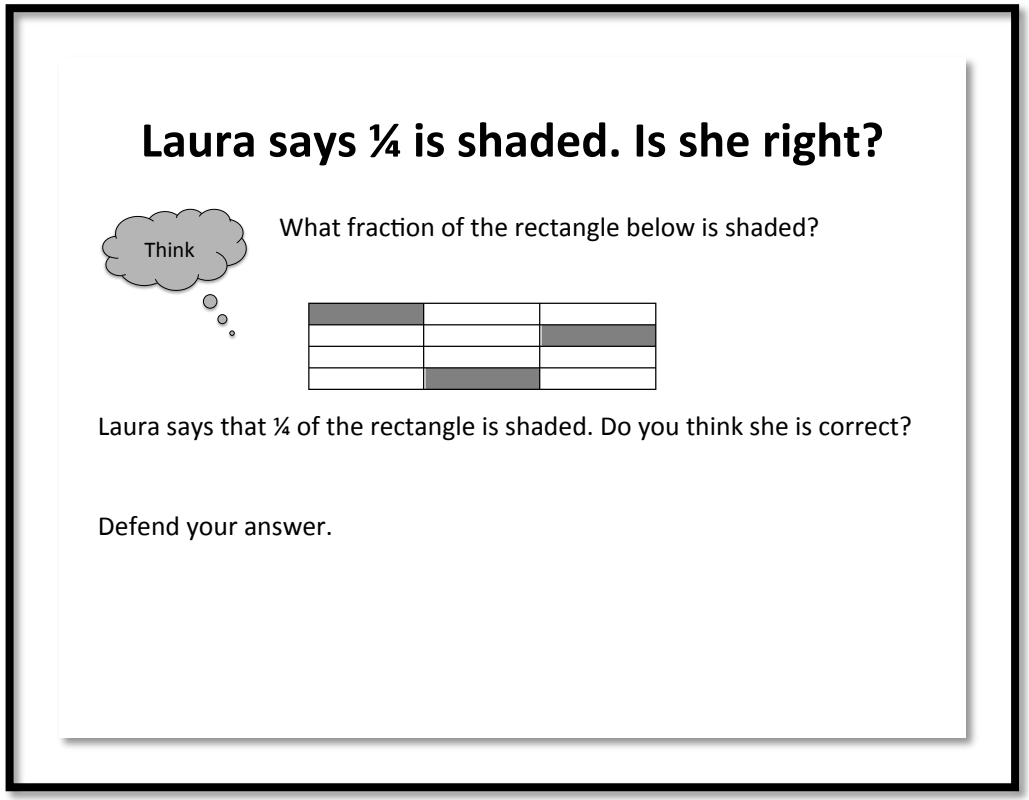
These criteria are helpful for discussions. It is important not to lose sight of the "big picture" however, and that is whether the argument offered shows that the claim is (or is not) true. This is the goal and purpose of a mathematical argument. You will see in many of these packets that students can approach an argumentation prompt from many different perspectives. It matters less *which* mathematical tools they use, and matters more whether their chain of reasoning compels the result.

In this packet you will find

1. A blank copy of the task: "Laura says 1/4 is shaded. Is she right?" and a description of the task implementation and/or other important considerations regarding student work samples included in this packet.
2. A protocol that can help you and your colleagues discuss student work related to this task. The use of the protocol is optional.
3. Selected work samples on this task from 4th-grade students in classes of teacher participants in the UConn Bridging Math Practices project to be used with the protocol.
4. Work Samples Classification and Commentaries: the student work samples ordered by whether they seem to be *high, adequate, or low quality* responses with respect to the criteria described on page 2 along with commentaries that support the classification. Among the samples are some that present a well-structured argument, but have important mathematical flaws, which prevent them from being classified as the highest quality.

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much "weight" people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional *discussions* and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

THE TASK



Source of the task: Adapted from Illustrative Mathematics https://www.illustrativemathematics.org/content-standards/tasks/881

CONTEXT

This problem was given to 4th-grade students at one school. The purpose of this problem was to see how students combine visual representation of fractions and conceptual understanding of equivalent fractions to justify their answer to the question in the problem. We offer one set of commentaries here. Student expectations may vary based on the manner in which the task is administered (time of the year, classroom norms, and prior knowledge about creating arguments). The student work samples in this set represent only a selection of the whole classroom work. It was apparent from the whole classroom set that students had been exposed to the idea that rearranging shaded parts on a visual representation does not change the value of the fraction it represents. Student work contained a variety of explicit and implicit warrants, and at times no warrants were presented.

Protocol Guided Sorting Activity: (33–40 mins)
Bridging Math Practices Math-Science Partnership Grant

This protocol was created for the purpose of reviewing student work. It is modified from two of the previously presented protocols in the Manchester School District. The original protocols apply to when teachers bring their own students' work. This has been modified to review prepared packets of student work.

- Maryland Protocol: Examining Student Work to Inform Instruction – Maryland State Department of Education http://mdk12.org/instruction/examining/protocol.html
- Collaborative Analysis Protocol - San Diego County Board of Education http://plc.sdcoe.net/Resources/Data%20Driven%20Decisions/LASWProtocol_Dec2011Rev.pdf

This is sometimes referred to as a ***Tuning Protocol,*** as the purpose is to help a group align their visions and expectations. Here, the alignment is with respect to the question: what is a high quality argument (on this task, for this grade level)? A main goal of this protocol is to support colleagues in building a consensus around what counts as a high quality argument.

0. Assign Roles

The Handler – places work samples in agreed-upon pile
Facilitator – ensures space is made for all to contribute; supports finding consensus
Time Keeper – keeps time and ensures group doesn't exceed section time limits. Can prompt movement to next section even if full time is not used.
All– share ideas and keep notes on own set of work samples

A: Setting the context for discussion (5 mins)

Team members read and do the problem. Team members discuss: What was the "big idea" of the task/assessment? What result or claim needed justification?

B: Quick sort: Reviewing student work (15 mins)

Do a *Quick Sort* of students' work by the degree of proficiency (high, adequate, low) demonstrated with providing an argument of the relevant claim(s). The Handler places a copy of the student work into the appropriate pile as agreed upon by the group. You may initially need a "Not Sure" pile. After sorting, revisit papers in the "Not Sure" pile and match each with the typical papers in one of the other piles. Record work sample numbers in the appropriate column of the chart (next page).

The facilitator may also decide to begin the Quick Sort with some silent review of student work samples before starting discussion.

Sorting Chart

HIGH Quality (high quality mathematical argument)	**ADEQUATE Quality** (adequate mathematical argument)	**LOW** (low quality mathematical argument)

C: Strengths and areas for growth? (5 mins)

Group member summarize key ideas from their Sorting Discussion regarding the strengths and areas for growth for individual samples, each group[1] (High Quality, Adequate, Low) of samples, or the overall set with respect to the argumentation?

HIGH Quality (high quality mathematical argument)	**ADEQUATE Quality** (adequate mathematical argument)	**LOW** (low quality mathematical argument)
Strengths overall for the class		

[1] This question is phrased in terms of "subgroups." You may or may not be able to characterize the group as a whole. As needed, describe individual or pairs of student work.

D: Reading ARP Commentaries (optional: 5-7 mins)

As deemed useful, group members read the commentaries in the Argumentation Resource Packet to gain new perspectives on selected student work samples, their strengths and areas for growth, and what counts as a high quality argument.

E: Reflection (5 mins) *Each person shares*

The facilitator guides the group to take turns in sharing a reflection. Group may decide to reflect on the same question, or each share a take away.

a. What did you learn about argumentation and how students engage argumentation from looking at the work of these students? You might also consider aspects of task design.

b. Did you have any *ah hah* moments?

c. What questions remain for you? What would you like to lean more about?

d. What will you take away from this discussion back to your classroom? What ideas might impact your planning or teaching?

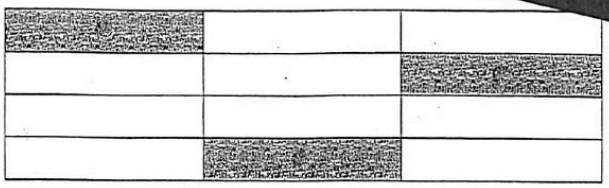
F: Reflection on Protocol Implementation (3 mins)

Facilitator guides a reflection on how the protocol process worked. Group members contribute ideas. Members make suggestions for modifications to future protocol as needed.

Student 1

Think

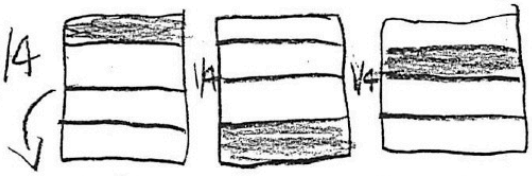
What fraction of the rectangle



Laura says that 1/4 of the rectangle is shaded. Do you think she is correct?

Defend your answer.

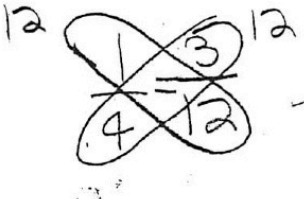
Yes, Laura is correct. She is because 1/4 is equal to 3/12.

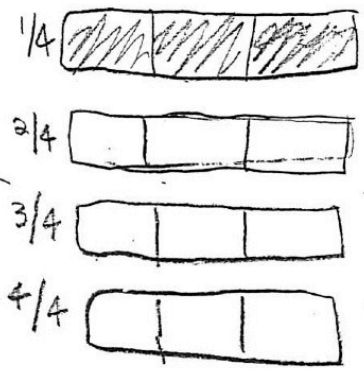


$\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$

This shows that the two fraction

are equal





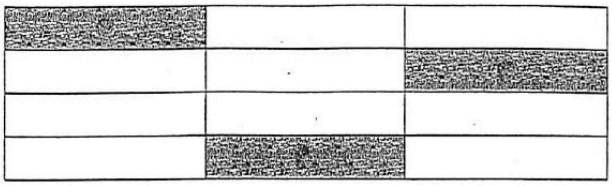
The tiles are just rearrange now it sho 1/4, so Lau is correct

I have shown four ways that the fractions are equal. This shows that Laura is correct.

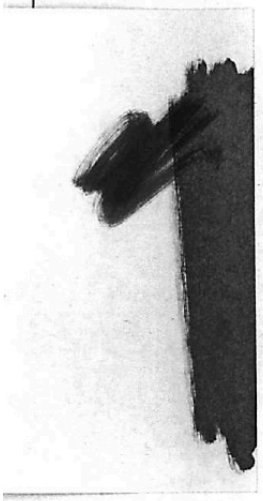
Student 2

Think

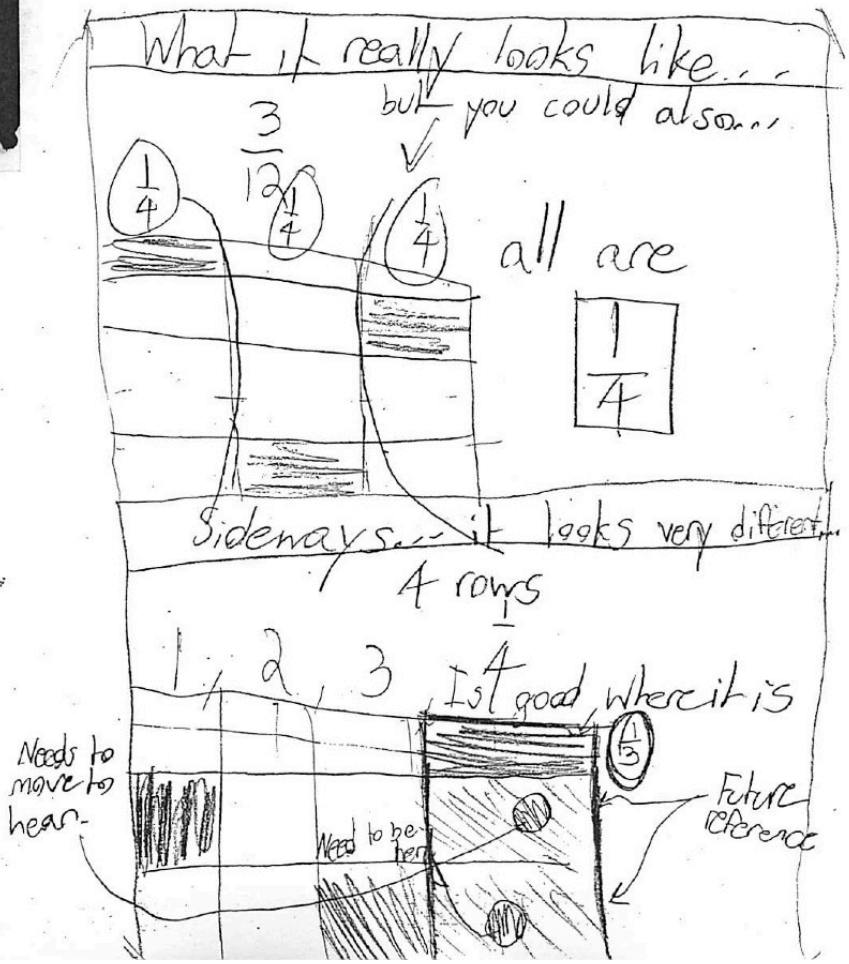
What fraction of the rectangle below is shaded?



Laura says that 1/4 of the rectangle is shaded. Do you think she is correct? Yes, I agree to Laura because of my work below.



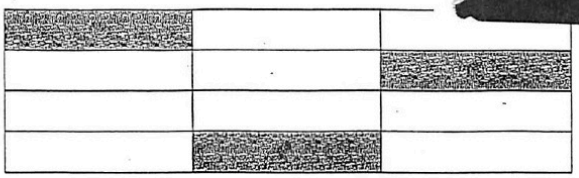
Defend your answer.



Student 3

Think

What fraction of the rectangle bel



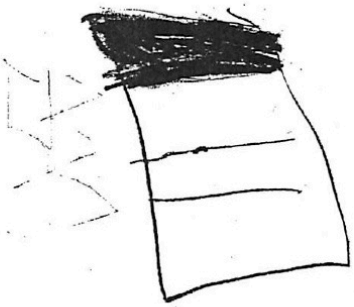
$\frac{3}{12}$

Laura says that 1/4 of the rectangle is shaded. Do you think she is correct?

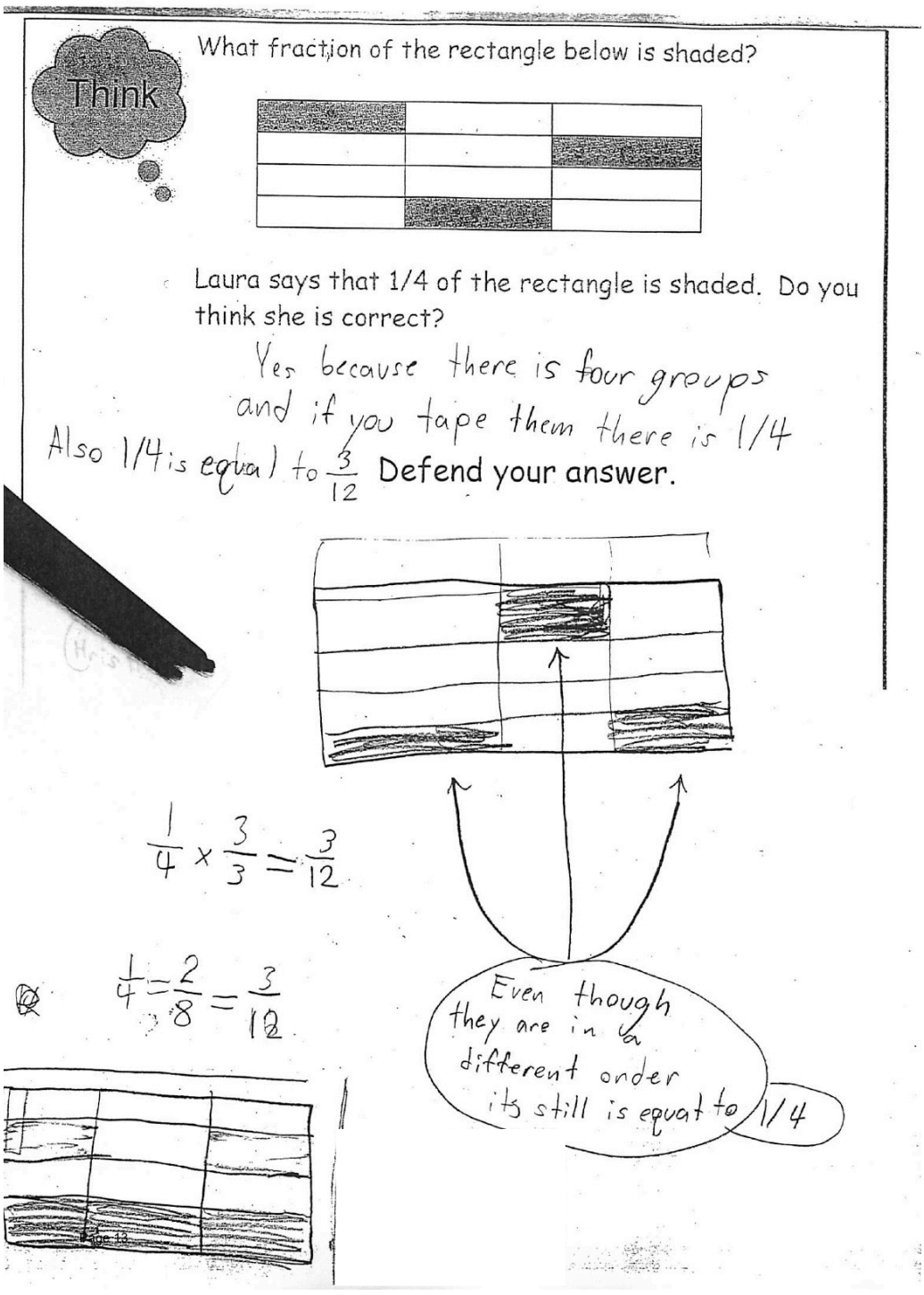
Defend your answer.

No laura 1 of the is right It is 1 of the row

$\frac{1}{4}$ of all together



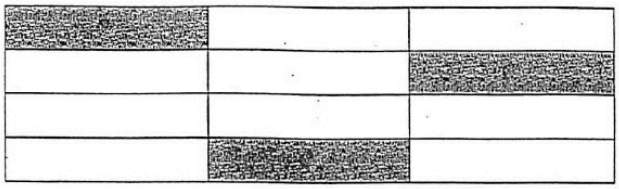
Student 4


Think
What fraction of the rectangle below is shaded?
Laura says that 1/4 of the rectangle is shaded. Do you think she is correct?
Yes because there is four groups and if you tape them there is 1/4
Also 1/4 is equal to 3/12 Defend your answer.
1/4 x 3/3 = 3/12
1/4 = 2/8 = 3/12
Even though they are in a different order its still is equal to 1/4

Student 5

Think

What fraction of the rectangle bel

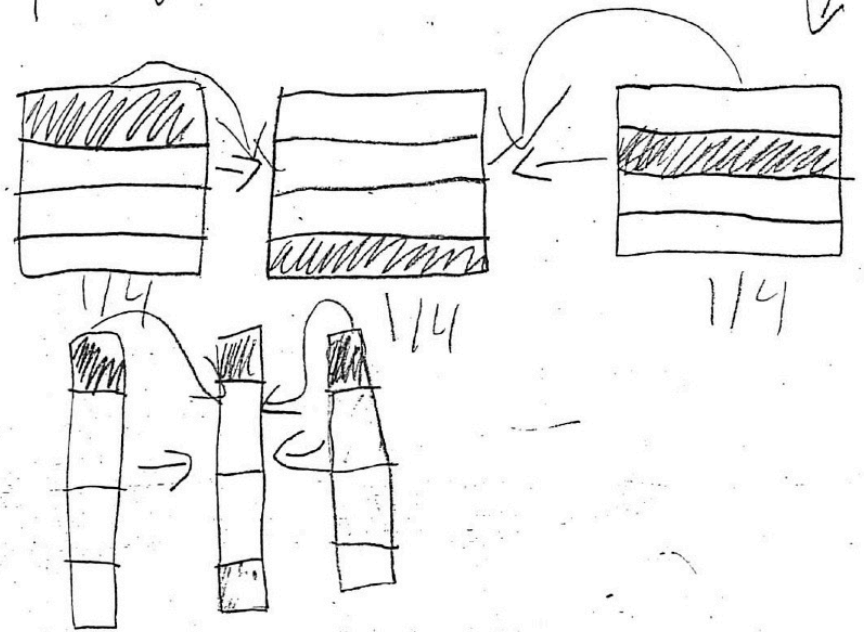


Laura says that 1/4 of the rectangle is shaded. Do you think she is correct?

Defend your answer.

Laura is correct because $\frac{3}{12}$ is equal to $\frac{1}{4}$. It shows me that Laura has 3 pieces of $\frac{1}{4}$ so Laura put the 3 pieces of $\frac{1}{4}$ together and she made $\frac{3}{12}$.

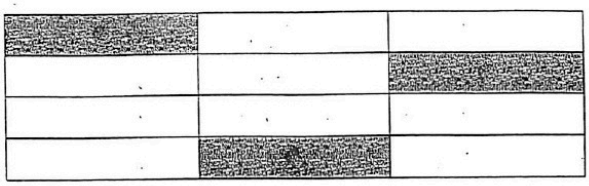
$\frac{3}{12} = \frac{1}{4}$



Student 6

Think

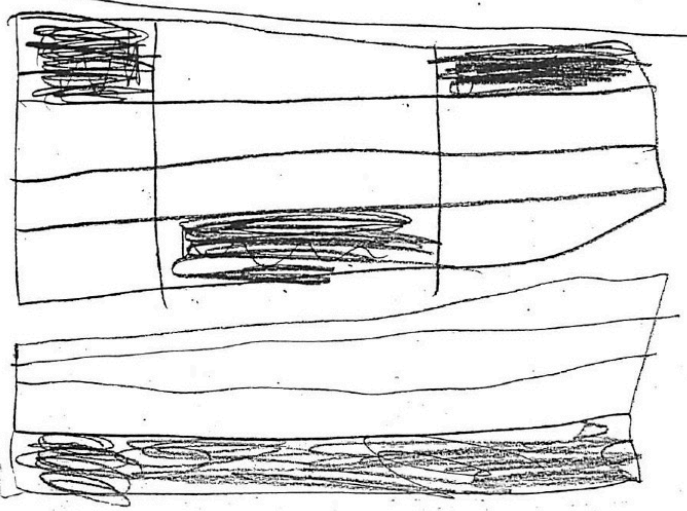
What fraction of the rectang [redacted]



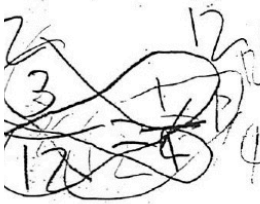
Laura says that 1/4 of the rectangle is shaded. Do you think she is correct?

Defend your answer.

Yes because 3/12 IS ekwivelint to 1/4 because if you get ride of the vertecile lines it ekwils 1/4.



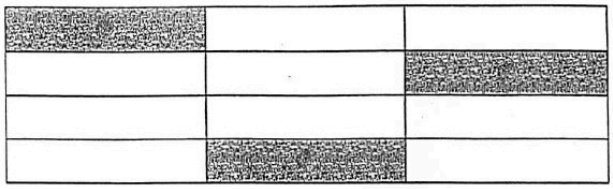
is = to



Student 7

Think

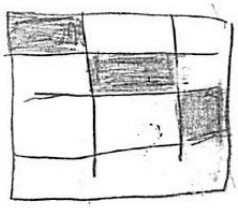
What fraction of the rectangle below is shaded?



Laura says that 1/4 of the rectangle is shaded. Do you think she is correct?

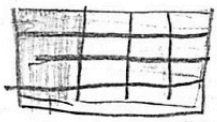
Defend your answer.

Yes I do or, think laura is correct because there is three Parts and each Part hav one Shaded bit if you rearange them like this  then it is still correct



Correct

LAuras



Correct

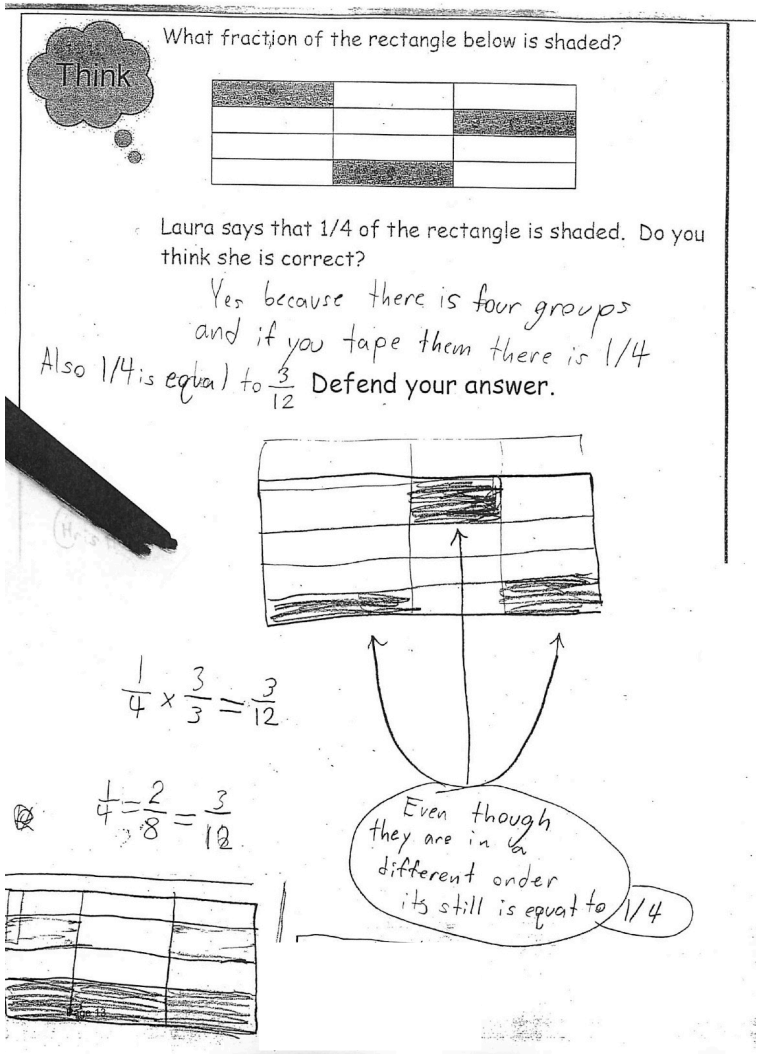
Work Samples Classification and Commentaries

Task: Laura Says ¼ is Shaded, Grade 4

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much "weight" people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional *discussions* and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

A Key linking the work samples from this ordered set with the sorting packet appears at the end of the document.

Student A



Commentary

This student's argument was categorized as **High Quality**.

Student A's claim is yes. The student identifies four groups (rows) and provides an argument to show that even though the shaded parts are in different rows, taken together ("tape") they ("still") equal ¼ of the rectangle. The student uses pictures and equations to show that ¼ is equivalent to 3/12.

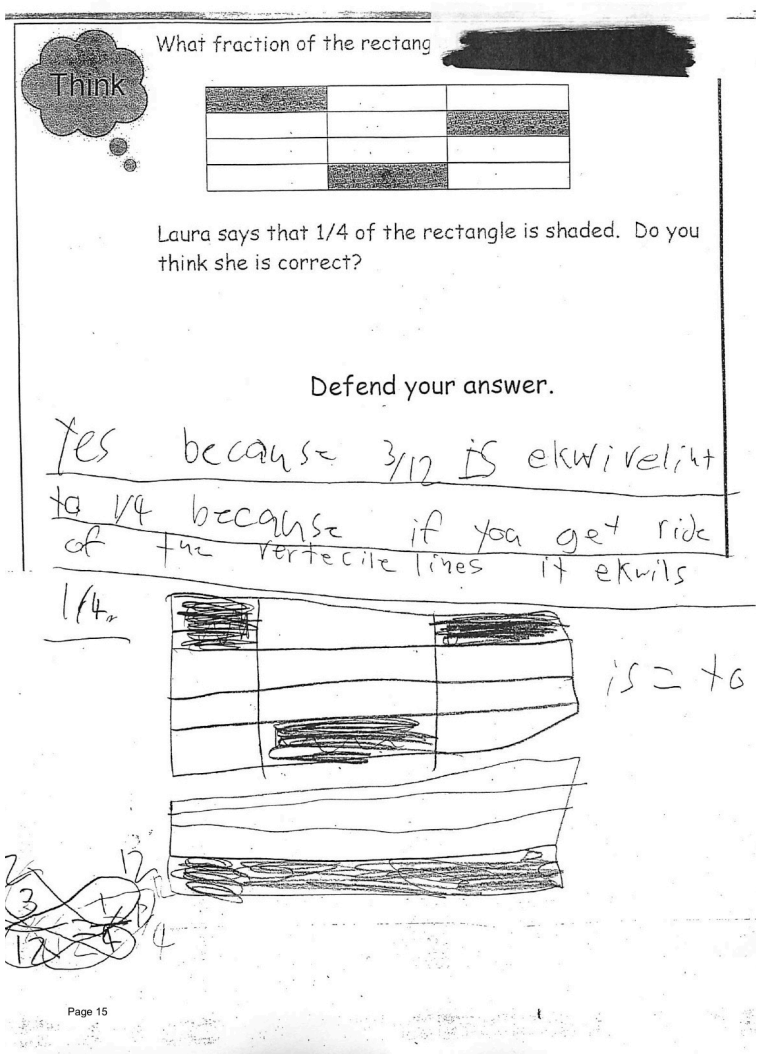
The student states that if you "tape" the shaded regions together (i.e., move them to the same row), they represent 1/4 of the rectangle. This is supported by the two pictures. The student used a multiplication sentence showing that when you multiply ¼ by 3/3, the result is an equivalent fraction: 3/12. The student also used a series of equivalent fractions confirming the equivalence.

This argument is considered high quality even though the warrant is weak. The response could be strengthened by improving the precision of language (i.e., use composed instead of taped or four rows instead of four groups) and expanding upon the warrant of ¼ = 3/12, by explaining, for example, why multiplying 3/3 creates an equivalent fraction. Notice also that the use of mathematical language is weak ("tape" and "groups"). Depending on the classroom norms, this could be considered a lower level argument.

Argumentation Components

Claim	**Evidence**
The claim is stated: Yes.	The student represents visually and in writing that 3/12 = ¼. In writing, the student used a series of equivalent fractions and a multiplication sentence to show that when you multiply ¼ by 3/3, the result is 3/12. The student also says if you "tape" the shaded regions together, it equals ¼.
Warrants	**Language & Computation**
The student states that ¼ = 3/12 to support the claim. The student also linked the pictures with the statement that even though the shaded parts on each picture are in different order, each shaded region is equal to ¼ of the rectangle.	The use of "tape" and "four groups" may be an accepted norm in the class, however, in general, the mathematical language used here would be considered weak.

Student B



Commentary

This student's argument was categorized as **High Quality**.

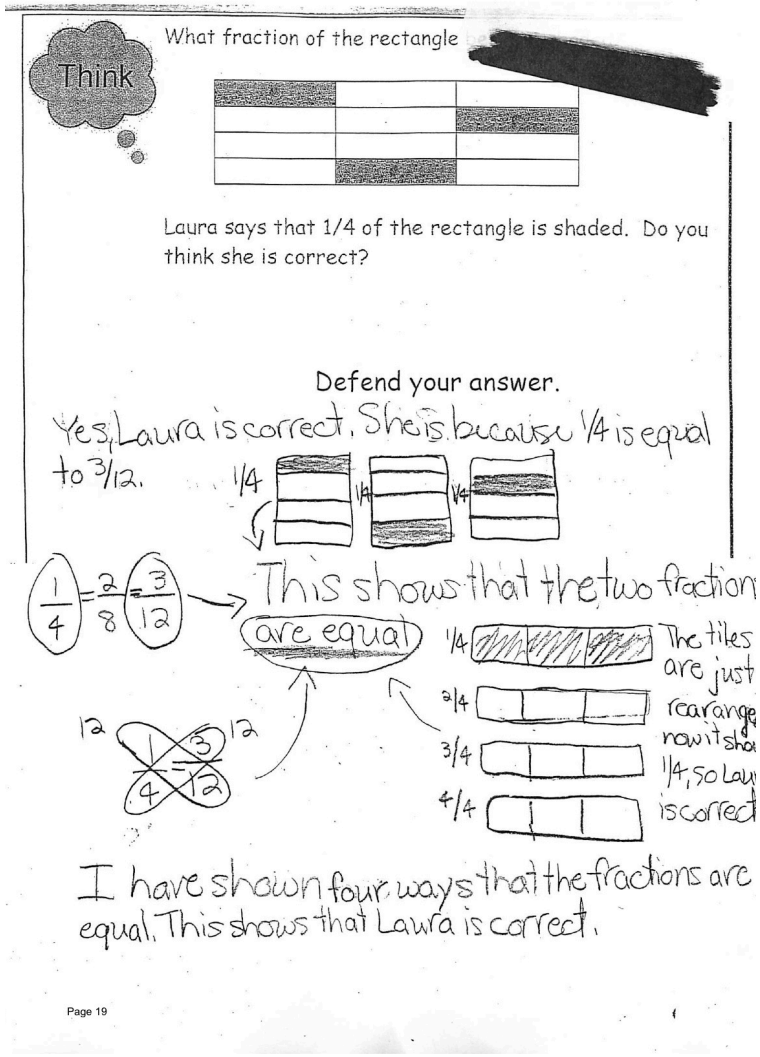
Student B's claim is yes. The student uses a model as evidence and an equation using cross multiplication to indicate reasoning that the three shaded parts represent ¼ of the whole. The student *implicitly* demonstrates understanding of how to interpret the value of a shaded region to represent equivalent fractions. Like Student A, Student B focuses on the rows (get rid of the vertical lines) and implies that moving all the shaded pieces down to the bottom row would create an area equivalent to ¼ but does not fully explain this rearrangement of the shaded pieces ("is = to").

The student uses adequate, although misspelled (e.g., ekwivelint), math language. Mathematical language could be expanded to explain the recomposing of the rectangle into four pieces instead of 12. It is recommended that spelling and symbolic issues be addressed at a later time and focus the assessment on the math argument.

Argumentation Components

Claim	Evidence
The claim is stated: Yes.	The student uses a visual representation supported by a statement about how removing vertical lines creates four equal rows that will have one row shaded. The student uses cross-multiplication apparently as a check for equivalency.
Warrants	**Language & Computation**
The warrant is stated as "3/12 is ekwivelint to ¼." This warrant is connected to the evidence by stating that removing the vertical lines creates a region of four pieces with one shaded.	The student uses adequate, although misspelled, math language to create understanding of the process of rearranging the pieces into one row.

Student C



Commentary

This student's argument was categorized as **High Quality**.

Student C's claim is, Yes, Laura is correct. Student C used multiple pictures, equations, and math language to show that ¼ is equivalent to 3/12.

The student used a series of equivalent fractions and a cross multiplication equation to show that ¼ = 3/12. The student also used a model to show visually (although partially inaccurately) that rearranging the shaded parts into one row more clearly shows the shaded parts equal ¼ of the whole rectangle.

The warrant is made stronger by the statement, "This shows that the two fractions are equal." This statement connects the evidence to the claim.

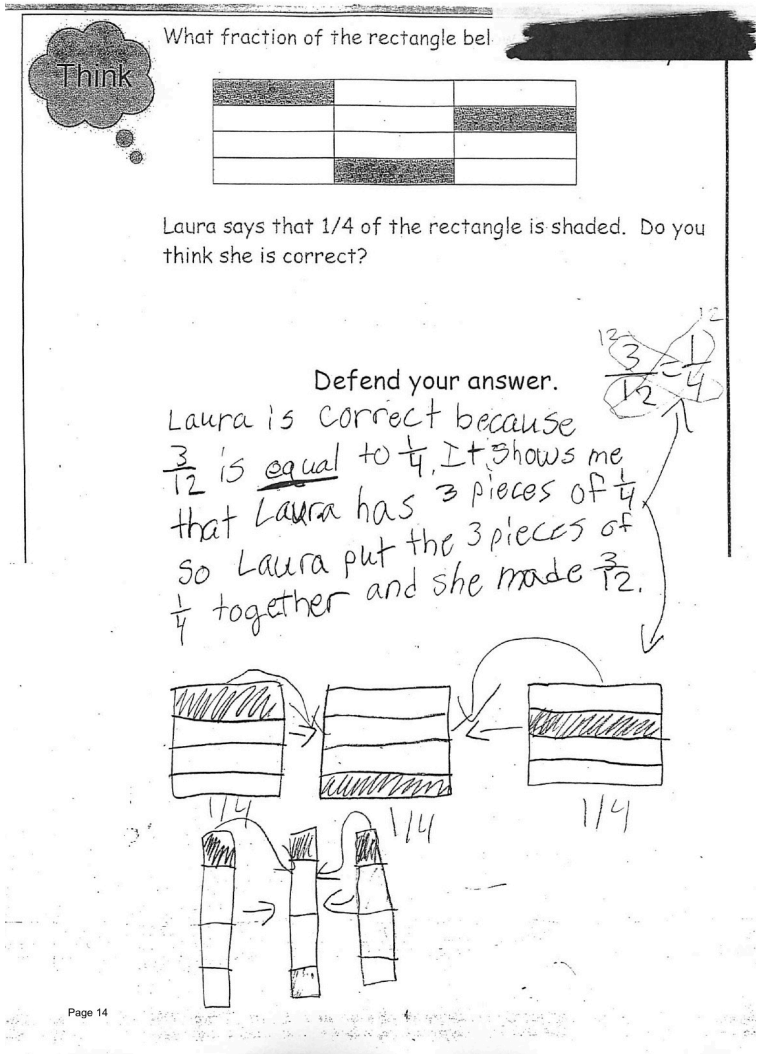
The labeling of the bottom picture is inaccurate. The first bar is labeled ¼, however the shaded parts represent 3/3, thus it is not clear what the label means. The next three bars are labeled 2/4, 3/4, and 4/4 respectively; however it is unclear how each of these represent those fractions. One can make different assumptions regarding what the thoughts of the student with this diagram, however, this is not accurately represented with the labels. The he last sentence seems to imply that arguments need to be supported in different ways; however, this is not necessary for mathematical arguments.

This argument is considered high quality because the student shows understanding of how to interpret a model to represent a fraction that is not immediately obvious.

Argumentation Components

Claim	**Evidence**
The claim is stated: Yes, Laura is correct.	The student represents visually and in writing that 3/12 = ¼. In writing, the student used a series of equivalent fractions and a cross-multiplication diagram to show that the two fractions render the same value of 12. The student
Warrants	**Language & Computation**
The warrant is stated as "1/4 is equal to 3/12." The warrant is supported in different ways that show the equivalence of the fractions.	The language used to support the argument is clear. As noted above, the labels on the bottom diagram are inaccurate.

Student D



Commentary

This student's argument was categorized as **Adequate Quality**.

Student D's claim is Laura is correct. The student uses a model (pictures and arrows) to show that joining all 1/4ths together results in the equivalent fraction 3/12.

Unlike Student C, the model shows combining/rejoining of the 3 pieces to a whole in which ¼ is shaded. The model implies rejoining, but the language could be more precise in explaining this process.

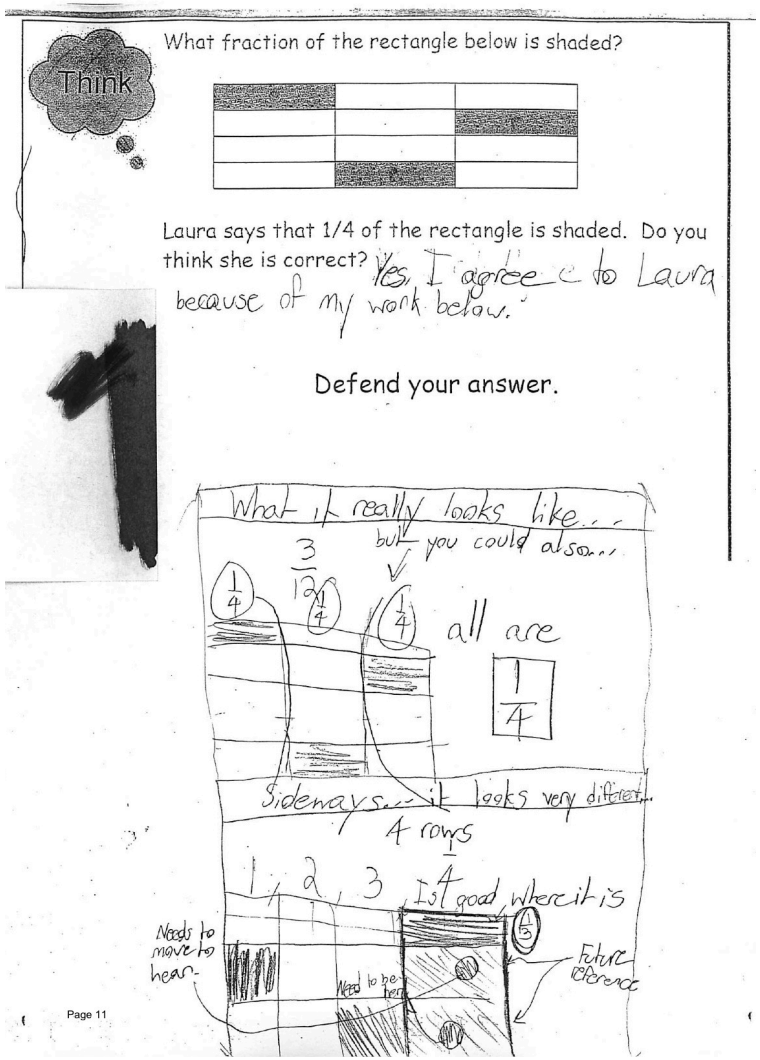
The argument does not make it clear why it can be concluded that the rejoined model also represents ¼ of the rectangle.

The student uses cross multiplication to show that the two fractions are equal.

Argumentation Components

Claim	**Evidence**
The claim is stated: Laura is correct.	The student uses a model (pictures and arrows) to show that joining all 1/4ths together result in the equivalent fraction 3/12. The student uses cross multiplication to verify equivalency.
Warrants	**Language & Computation**
Student's main support to the claim is that 3/12 is equal to ¼. Student also links model to statement explaining that three 1/4ths, when considered together, represent 3/12, but the explanation is incomplete.	The language and calculations used are correct; however, the argument would be stronger with an explanation of what the arrows between the diagram represent and appropriate warrants for that were offered.

Student E



Commentary

This student's argument was categorized as **Adequate Quality**.

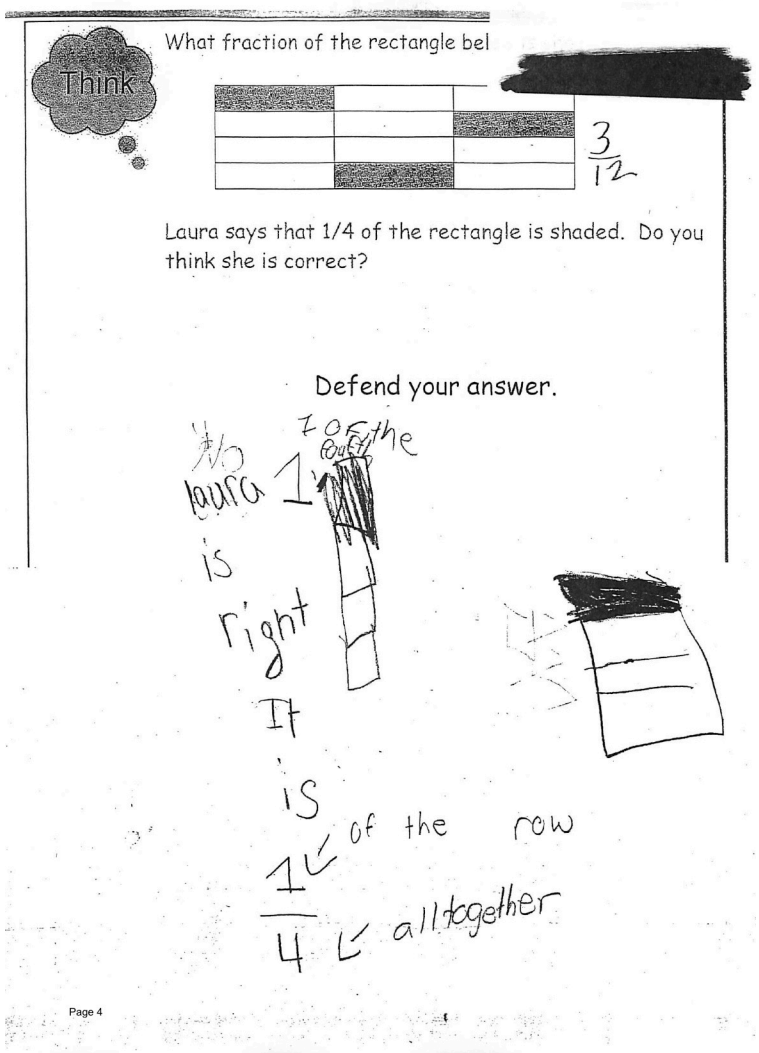
Student E's claim is *yes*. The student initially redrew the rectangle, showed that the shaded region represents 3/12, and then indicated ¼ is in each one of the columns, "All are ¼." Student E then redrew the model sideways and showed that if you move two of the shaded parts so that all three shaded parts are in one column, it is easier to see the ¼ of the whole rectangle. By numbering the columns, it is implied that 1 of the four columns is shaded and therefore equal to ¼.

The student seems to understand that rearranging the shaded parts does not change the value of the fraction. The argument would be stronger if the student offered a warrant explicitly addressing the equivalency of 3/12 and ¼ along with more precise mathematical vocabulary, such as whole, parts, and regions.

Argumentation Components

Claim	**Evidence**
The claim is stated: Yes.	Evidence is provided in the form of a new diagram in which the original has been redrawn by moving its shaded parts to show that the same diagram can also be seen as 1 column shaded out of 4.
Warrants	**Language & Computation**
The warrant used is that when the diagram represents the same fractions if looked at from a different perspective or moving the shaded parts to other places on the diagram.	The language and computations used is considered sufficient to follow the argument. None the less, the argument would be strengthened with more explanation and better vocabulary use (whole, parts, regions)

Student F



Commentary

This student's argument was categorized as **Low Quality**.

Student F's claim is Laura is right. Student F identifies the original picture has 3/12 shaded. The student also attempts to use a model to show that each column is 1/4; however, there is no clear evidence of equality between ¼ and 3/12. There are no warrants to link the beginnings of evidence to the claim.

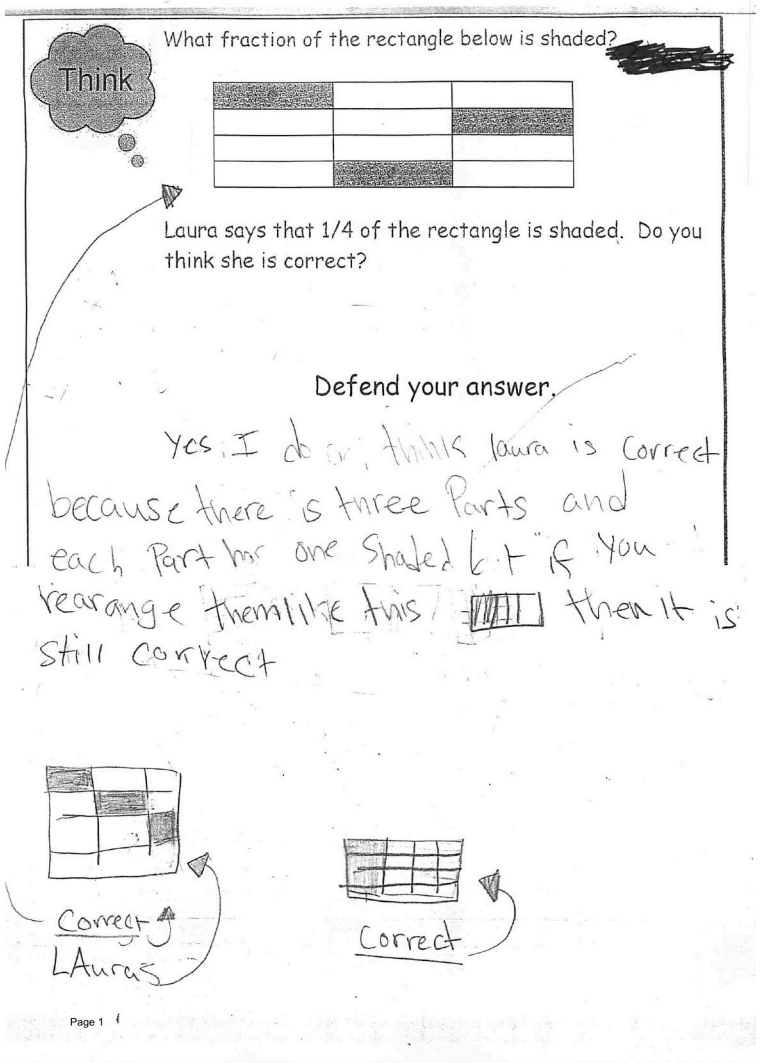
Other explanations for the student's work may be possible, but would require making many inferences.

The student does not use precise vocabulary to clearly communicate the ideas that support the argument.

Argumentation Components

Claim	**Evidence**
The claim is stated: Laura is right.	Student identifies given model as 3/12 and uses a model that shows ¼th. However, it is difficult to interpret the connection between the student's model and the claim.
Warrants	**Language & Computation**
The explanation offered by the student to link the model to the claim is weak and hard to interpret: "1 of the row" and "4 altogether".	Vocabulary needs to be strengthened to make the argument clearer. For example, it is not clear how to interpret "alltogether" in this fractions context.

Student G



Commentary

This student's argument was categorized as **Low Quality**.

Student G's claim is yes. Student G used a model to show that the value of the fraction does not change even though the arrangement is different.

Student G's response does not include precise language (e.g., does not explain what "parts" are). The models could be improved if drawn with mathematical accuracy (e.g., it is unclear if one of the models shows a 4x4 or 4x3 grid).

The argument would be stronger if the student provided the explanation that if you divide the whole into three smaller wholes, and each whole has ¼ shaded, then the combined shaded area will also represent ¼.

The student shows fair understanding but fails to make a fluent argument. The student does not use precise vocabulary to clearly communicate the ideas that support the argument.

Argumentation Components

Claim	**Evidence**
The claim is stated: Yes.	The student writes the statement "there is three parts and each part has one shaded…" and a visual to show that rearranging the parts of the whole does not change the value of the fractional part shaded.
Warrants	**Language & Computation**
The student does not use warrants to link the claim and the evidence.	The student's language lacks precision. For example, when writing "three parts" without being specific about which three parts. The student does not use mathematical vocabulary such as equivalent, equal to, or value.

Key Connecting Sorting Packet to Argumentation Resource Packet

Student number (Sorting Packet)	Resource Packet Sample
1	C (high)
2	E (adequate)
3	F (low)
4	A (high)
5	D (adequate)
6	B (high)
7	G (low)

Student number (Sorting Packet)	Resource Packet Sample (category)
4	A (high)
6	B (high)
1	C (high)
5	D (adequate)
2	E (adequate)
3	F (low)
7	G (low)