

## DJ for the Prom Problem (Algebra I)

### STUDENT WORK SAMPLE ARGUMENTATION RESOURCE PACKET



This packet was produced as part of the Bridging Math Practices Math-Science Partnership Grant (2014 -2015).

The purpose of the packet is to help a) reveal what students can do with respect to generating an argument in response to mathematical questions, including the variety of their arguments; b) highlight features that should be considered when reviewing students' arguments, and c) identify what counts as a *quality* argument in light of the review criteria.

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What is a mathematical argument?

A mathematical argument is

a sequence of statements and reasons given with the aim of demonstrating that a claim is true or false.

This links to the Connecticut Core Standards of Mathematical Practice #3, *construct viable arguments and critique the reasoning of others*, as well as other standards.

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This resource packet is a product of work by participants in the UConn Bridging Math Practices Math-Science Partnership Grant, which included faculty and graduate students from the University of Connecticut's Neag School of Education and Department of Mathematics, and teachers and coaches from the Manchester Public Schools, Mansfield Public Schools, and Hartford Public Schools. This resource packet reflects significant contributions from Jenn Downes, Jeana Favat, Cathy Mazzotta, Belinda Perez, Adrienne Satin, and John Tedesco. Many thanks for all their insights and contributions! For more information about the grant, or for additional argumentation-related materials and resource, please see the project website: <http://bridges.uconn.edu>

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## What is a high quality mathematical argument?

A high quality mathematical argument is an argument that shows that a claim must be true. It leaves little room to question. The chain of logic leads the reader to conclude that the author's claim is true.

What are the characteristics of a high quality argument? A high quality argument can be described by the following components and criteria:

Criteria	Description
1. A clearly stated <b>claim</b> 2.	The claim is what is to be shown true or not true.
2. The necessary <b>evidence</b> to support the claim	Evidence can take the form of equations, tables, charts, diagrams, graphs, words, symbols, etc. It is one's "work" which provides the information to show something is true/false.
3. The necessary <b>warrants</b> to connect the evidence to the claim	Warrants can take the form of definitions, theorems, logical inferences, agreed upon facts. Warrants explain how the evidence is relevant for the claim, and collectively they chain the evidence together to show the claim is true or false.
4. <b>Language use and computations</b> are at a sufficient level of precision and accuracy	The language used and computations must be at a sufficient level of precision or accuracy to support the argument. Language use needs to be precise enough to communicate the ideas with sufficient clarity.

These criteria are helpful for discussions. It is important not to lose sight of the "big picture" however, which is whether the argument offered shows that the claim is (or is not) true. This is the goal and purpose of a mathematical argument. You will see in many of these packets that students can approach an argumentation prompt from many different perspectives. It matters less *which* mathematical tools they use, and matters more whether their chain of reasoning compels the result.

In this packet you will find

1. A blank copy of the task (DJ for the prom) and a description of the task implementation and/or other important considerations regarding student work samples included in this packet.
2. A protocol that can help you and your colleagues discuss student work related to this task. The use of the protocol is optional.
3. Selected work samples on this task from high school students (grades 9 and 10, across courses) in a school participating in the UConn Bridging Math Practices project to be used with the protocol.
4. Work Samples Classification and Commentaries: the student work samples ordered by whether they seem to be high, adequate, or low quality responses with respect to the criteria described on page 2 along with commentaries that support the classification. Among the samples are some that present a well-structured argument, but have important mathematical flaws, which prevent them from being classified as the highest quality.

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much “weight” people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional discussions and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

## THE TASK

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### DJ for the Prom

In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$ 175per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

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### CONTEXT

This problem was given to all 9<sup>th</sup>- and 10<sup>th</sup>-grade students. These students are from a variety of levels and classes ranging from Algebra 1 to Pre-calculus. The purpose of this problem was to see not only how students solve, but also reason through a common type of algebraic task. Student expectations may vary based on the class and manner in which it is administered (Algebra 1 vs. Algebra 2 or Geometry). Given the variety of students, implied warrants were at times difficult to determine because there were differences across classes in previously agreed upon background knowledge and established theorems and definitions. It was, however, a (practice) assessment situation, and so we expected students to be more explicit than they might be on a class assignment. The responses in the low and adequate categories are ranked from strongest to weakest argument in that category. In the high category, all responses are considered equally strong, but were chosen to highlight the students' use of different methods.

## Protocol Guided Sorting Activity: (33–40 mins)

### Bridging Math Practices Math-Science Partnership Grant

This protocol was created for the purpose of reviewing student work. It is modified from two of the previously presented protocols in the Manchester School District. The original protocols apply to when teachers bring their own students' work. This has been modified to review prepared packets of student work.

- Maryland Protocol: Examining Student Work to Inform Instruction – Maryland State Department of Education <http://mdk12.org/instruction/examining/protocol.html>
- Collaborative Analysis Protocol - San Diego County Board of Education [http://plc.sdcoe.net/Resources/Data%20Driven%20Decisions/LASWProtocol\\_Dec2011Rev.pdf](http://plc.sdcoe.net/Resources/Data%20Driven%20Decisions/LASWProtocol_Dec2011Rev.pdf)

This is sometimes referred to as a ***Tuning Protocol***, as the purpose is to help a group align their visions and expectations. Here, the alignment is with respect to the question: what is a high quality argument (on this task, for this grade level)? A main goal of this protocol is to support colleagues in building a consensus around what counts as a high quality argument.

## 0. Assign Roles

**The Handler** – places work samples in agreed-upon pile

**Facilitator** – ensures space is made for all to contribute; supports finding consensus

**Time Keeper** – keeps time and ensures group doesn't exceed section time limits. Can prompt movement to next section even if full time is not used.

**All**– share ideas and keep notes on own set of work samples

## A: Setting the context for discussion (5 mins)

Team members read and do the problem. Team members discuss: What was the “big idea” of the task/assessment? What result or claim needed justification?

## B: Quick sort: Reviewing student work (15 mins)

Do a *Quick Sort* of students' work by the degree of proficiency (high, adequate, low) demonstrated with providing an argument of the relevant claim(s). The Handler places a copy of the student work into the appropriate pile as agreed upon by the group. You may initially need a “Not Sure” pile. After sorting, revisit papers in the “Not Sure” pile and match each with the typical papers in one of the other piles. Record work sample numbers in the appropriate column of the chart (next page).

*The facilitator may also decide to begin the Quick Sort with some silent review of student work samples before starting discussion.*

### Sorting Chart

<b>HIGH Quality</b> (high quality mathematical argument)	<b>ADEQUATE Quality</b> (adequate mathematical argument)	<b>LOW</b> (low quality mathematical argument)

### C: Strengths and areas for growth? (5 mins)

Group member summarize key ideas from their Sorting Discussion regarding the strengths and areas for growth for individual samples, each group<sup>1</sup> (High Quality, Adequate, Low) of samples, or the overall set with respect to the argumentation?

<b>HIGH Quality</b> (high quality mathematical argument)	<b>ADEQUATE Quality</b> (adequate mathematical argument)	<b>LOW</b> (low quality mathematical argument)
<b>Strengths overall for the class</b>		

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<sup>1</sup> This question is phrased in terms of “subgroups.” You may or may not be able to characterize the group as a whole. As needed, describe individual or pairs of student work.

### **D: Reading ARP Commentaries (optional: 5-7 mins)**

As deemed useful, group members read the commentaries in the Argumentation Resource Packet to gain new perspectives on selected student work samples, their strengths and areas for growth, and what counts as a high quality argument.

### **E: Reflection (5 mins) *Each person shares***

The facilitator guides the group to take turns in sharing a reflection. Group may decide to reflect on the same question, or each share a take away.

- a. What did you learn about argumentation and how students engage argumentation from looking at the work of these students? You might also consider aspects of task design.
- b. Did you have any *ah hah* moments?
- c. What questions remain for you? What would you like to learn more about?
- d. What will you take away from this discussion back to your classroom? What ideas might impact your planning or teaching?

### **F: Reflection on Protocol Implementation (3 mins)**

Facilitator guides a reflection on how the protocol process worked. Group members contribute ideas. Members make suggestions for modifications to future protocol as needed.

# Student 1

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

Music Makers : \$200 + \$175 per hour

$$2 \text{ hours} = \$550$$

$$3 \text{ hours} = \$750$$

$$4 \text{ hours} = \$900$$

$$6 \text{ hours} = \$1,250$$

$$8 \text{ hours} = \$1,600$$

Dance Partners : \$225 per hour

$$2 \text{ hours} = \$450$$

$$3 \text{ hours} = \$675$$

$$4 \text{ hours} = \$900$$

$$6 \text{ hours} = \$1,350$$

$$8 \text{ hours} = \$1,800$$

Depending on how many hours you would have the dj for would effect your decision. If you were to have the dj for only 2-3 hours, Dance Partners would be the cheaper choice, yet if you wanted the dj for 4 + hours, music makers would be the best choice.



## Student 2

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$ 175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

explain

An average prom lasts 4 hours. Assuming the prom would last between 1 and 4 hours, a chart comparing the cost of each DJ companies with the length of the prom is made below

	Music Makers	Dance Partners
1 hour	\$ 375	\$ 225
2 hour	\$ 550	\$ 450
3 hour	\$ 725	\$ 675
4 hour	\$ 900	\$ 900

- ① ~~\$200 + 175~~ Equation for music makers =  $175x + 200 = y$   
Equation for DP =  $225x = y$

Answer = If the prom was 4 hours long, ~~either~~ both DJ companies would cost the same. However, anytime less than 4 hours would result in Dance Partners being the most cost effective, as they are cheaper each hour.

## Student 3

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

MUSIC MAKERS - \$200 + \$175 per hour

Dance partners - \$225 per hour

	MM	DP
1	375	225
2	550	450
3	725	675
4	900	900

It depends how long prom will last to see which DJ is the most cost effective. From one hour to three hours, dance partners would be cheaper. But if prom was four hours, they would cost the same amount. And, if prom was more than 4 hours, Music Makers would be cheaper.

## Student 4

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

$$\begin{array}{r} \text{DJ A} \\ \hline \text{Fee} - 200\$ \\ + \\ 175 \times \text{hours} \end{array}$$

$$\begin{array}{l} 1 \text{ hour} = 375 \\ 2 \text{ hour} = 575 \\ 3 \text{ hour} = 750 \\ 4 \text{ hours} = 925 \\ 5 \text{ hours} = 1100 \end{array}$$

$$\begin{array}{r} \text{DJ B} \\ \hline \text{fee} - 0\$ \\ 225 \text{ hour} \end{array}$$

$$\begin{array}{l} 1 \text{ hour} = 225 \\ 2 \text{ hour} = 450 \\ 3 \text{ hours} = 675 \\ 4 \text{ hours} = 900 \\ 5 \text{ hours} = 1125 \end{array}$$

Dance Partners is most effective because after 3 hours of DJ the cost was only 675.00\$ unlike Music Makers who charged 750.00\$. Music Makers always cost more until 5 hours of DJ when the cost was 1100.00\$ and Dance Partners was 1125.00\$.

## Student 5

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

Even though Music Makers starts with a higher fee because they do \$200 plus \$175 per hour, Dance Partners will eventually be more expensive because they have a bigger hourly rate.

Dance Partners is \$50 more per hour.

So it takes 4 hours to catch up and

$$50 + 50 + 50 + 50 = 200.$$

make up the initial fee.

At 4 hrs, they are the same.

After that, MM is less because the hourly rate is less.

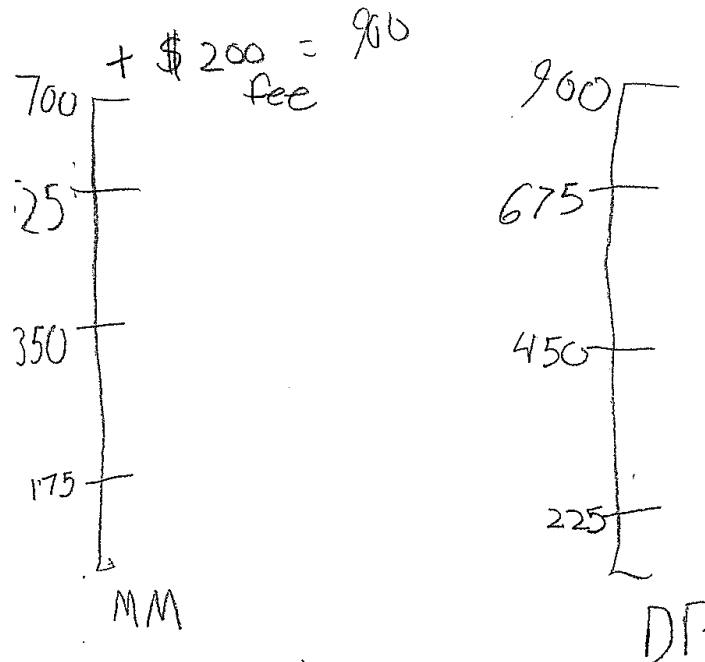
4 hrs - either one

< 4 hrs - Dance Partner

> 4 hrs - Music Makers.

# Student 6

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.



Both music companies would charge you the same price for the prom.

## Student 7

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

	Start		Per hour
Music Makers:	\$200	+	\$175
Dance partners:	\$ initial		\$225

$$x = 5 \quad 200 + 175x = 1075$$
$$225x = 1125$$

I think that Dance partners DJ company would be more cost effective. I think this because no matter the amount of hours ( $x$ ) it will still cost more than Music Makers.

## Student 8

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

$$200 + 175x = C$$

$$225x = C$$

$$225(5) = 1125$$

$$200 + 175(5) = 1075$$

$$\begin{array}{r} 200 + 175x = 225x \\ -175x \quad 175x \\ \hline \end{array}$$

$$\begin{array}{r} 200 = 50x \\ 50 \quad 50 \\ \hline \end{array}$$

$$4 = x$$

Music Makers is more cost effective if Prom is longer than 4 hours. I know this because the 2 DJs cost the same amount for 4 hours and Music makers cost less when the fifth hour hit, but Dance Partners cost less if Prom is less than 4 hours.

## Student 9

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

$$\begin{array}{l} \text{Music makers: } 175x + 200 \\ \text{Dance partners: } 225x \end{array} \quad (\text{plus in})$$

Answer: Music Makers will be more cost effective if IF you rent this company for over 4 hours. But if you rent the group for only four hours, they both will be good because they both cost \$900 at 4 hours. But if you rent it for less than 4 hours, Dance Partners would be better.



# Student 10

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

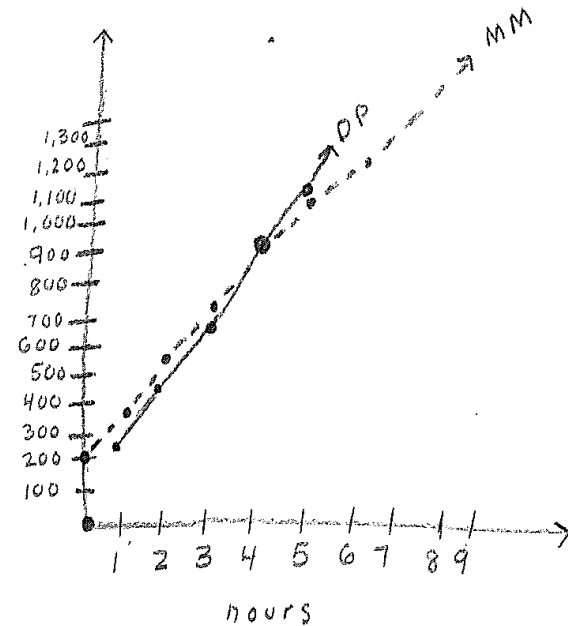
$$MM = 175x + 200$$

$$DP = 225x$$

$$\begin{array}{r} 225x = 175x + 200 \\ -175x \quad -175x \\ \hline 50x = 200 \end{array}$$

$$\begin{array}{r} 50x = 200 \\ \hline 50 \quad 50 \\ \hline x = 4 \end{array}$$

$$x = 4$$



	MM	DP
0	200	0
1	375	225
2	550	450
3	725	675
4	900	900
5	1075	1125
6	1250	1350

If the prom will last less than 4 hours they should go with Dance Partners if it will last more than 4 hours they should go with music makers, if it will last exactly 4 hours it will not matter because it will be the same price

# **DJ for the Prom Problem (Algebra I)**

## **ANNOTATED**

### **STUDENT WORK SAMPLE**

### **ARGUMENTATION RESOURCE**

### **PACKET**

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much “weight” people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional discussions and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

# Student 2

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

*Explanation*  
An average prom lasts 4 hours. Assuming the prom would last between 1 and 4 hours, a chart comparing the cost of each DJ company with the length of the prom is made below.

	Music Makers	Dance Partners
1 hour	\$375	\$225
2 hour	\$550	\$450
3 hour	\$725	\$675
4 hour	\$900	\$900

- ① ~~Equation~~ Equation for music makers =  $175x + 200 = y$   
Equation for DP =  $225x = y$

Answer = If the prom was 4 hours long, ~~either~~ both DJ companies would cost the same. However, anytime less than 4 hours would result in Dance Partners being the most cost effective, as they are cheaper each hour.

## Commentary

This student's argument was categorized as **HIGH quality**.

The student claims that both DJ companies cost the same for "an average prom" of 4 hours. The evidence to support the claim is the table created for finding costs for 1 to 4 hours. The student provides an implicit warrant by setting up linear equations to model the cost of hiring each company for  $x$  hours, and provides an explicit warrant by directly comparing the costs for the prom that is on average 4 hours long.

Note that this student states explicitly that the question s/he was answering was about an average prom, which the student reasonably assumed to be about 4 hours. The justification offered fully addresses this question of the 4-hour prom. Given that the student is asked to engage this as a "real world" problem, adding "real world" constraints is appropriate, provided the constraints are reasonable and explicitly stated.

# Student 10

## Commentary

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

$$MM = 175x + 200$$

$$DP = 225x$$

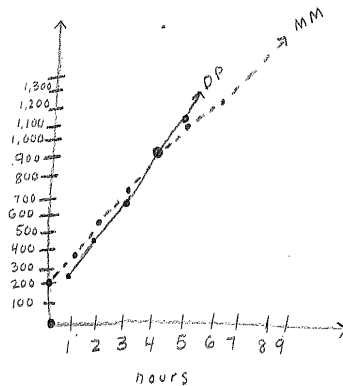
$$225x = 175x + 200$$

$$-175x \quad -175x$$

$$50x = 200$$

$$50 \quad 50$$

$$x = 4$$



	MM	DP
0	200	0
1	375	225
2	550	450
3	725	675
4	900	900
5	1075	1125
6	1250	1350

If the prom will last less than 4 hours they should go with Dance Partners, if it will last more than 4 hours they should go with music makers, if it will last exactly 4 hours it will not matter because it will be the same price

This student's argument was categorized as **HIGH quality**.

The student claims that the company that is more cost effective depends on the length of time. The student clearly explains all 3 cases. For evidence, the student provides multiple representations of evidence – a table, graph showing the point of intersection, equations with calculations deriving  $x = 4$  (the result of setting the equations equal and solving). (Note: Not all representations are needed to support this claim. One would be sufficient.) The warrant is implicit, as the student leaves to the reader the explanation of how each of these representations shows that the costs of the two companies are equal at 4 hours. The table perhaps is the most obvious of these, with its direct comparison of the costs of both companies side-by-side.

The response could be strengthened further by using just *one* representation, and by explaining more clearly the connection between the representation and the claim.

# Student 5

1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

Even though Music Makers starts with a higher fee because they do \$200 plus \$175 per hour, Dance Partners will eventually be more expensive because they have a bigger hourly rate.

Dance Partners is \$50 more per hour.

So it takes 4 hours to catch up and  
 $50 + 50 + 50 + 50 = 200$   
make up the initial fee.

At 4 hrs, they are the same.

After that, MM is less because the hourly rate is less.

4 hrs - either one

< 4 hrs - Dance Partner

> 4 hrs - Music Makers.

## Commentary

This student's argument was categorized as **HIGH quality**.

The student claims that which company is more cost effective depends on the length of prom and clearly explains all 3 cases.

The evidence that the two companies cost the same for four hours is based on the \$50 differential in their hourly rate and showing that  $50 + 50 + 50 + 50 = 200$ . The warrant is based on the idea of "closing the gap:" if each hour there is a \$50 difference, and there is an initial \$200 difference, then it takes 4 hours to close the gap – that is, for Dance Partners to "catch up" with Music Makers.

The evidence to support the claim that Music Makers is more expensive after 4 hours is based on the hourly rates: "Dance Partners is more per hour" and MM's "hourly rate is less." The warrant is implicit: for any number of hours after the costs are equal the company with the lower hourly rate will costs less.

This argument could be strengthened by making more explicit the warrants, which could be explained more clearly.

# Student 3

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

Music Makers - \$200 + \$175 per hour  
Dance Partners - \$225 per hour

	MM	DP
1	375	225
2	550	450
3	725	675
4	900	900

It depends how long prom will last to see which DJ is the most cost effective. From one hour to three hours, dance partners would be cheaper. But if prom was four hours, they would cost the same amount. And, if prom was more than 4 hours, music makers would be cheaper.

## Commentary

This student's argument was categorized as **ADEQUATE quality**.

The student claims that it depends on how long prom will be and that there are 3 cases. The student provides evidence for 2 of the 3 cases (using the table), and does not provide evidence or reasoning for what happens after the costs are the same. The warrant that supports the claim is directly comparing the costs by the addition of the initial fees and costs per hour.

The argument could be strengthened if it included evidence for the cost after 4 hours, or make an argument that once the total costs is equal at 4 hours, for any prom that's longer, Dance Partners will cost more as it has a higher hourly rate.

# Student 8

1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

$$200 + 175x = C$$

$$225x = C$$

$$225(5) = 1125$$

$$200 + 175(5) = 1075$$

$$\begin{array}{r} 200 + 175x = 225x \\ -175x \quad -175x \\ \hline 200 = 50x \\ 50 \quad 50 \\ \hline 4 = x \end{array}$$

Music Makers is more cost effective if Prom is longer than 4 hours. I know this because the 2 DJs cost the same amount for 4 hours and Music makers cost less when the fifth hour hit, but Dance Partners cost less if Prom is less than 4 hours.

## Commentary

This student's argument was categorized as **ADEQUATE quality**.

The student claims that Music Makers (MM) is more cost effective if prom is longer than 4 hours. The student provides evidence for when the two companies cost the same (setting two equations equal to each other). The student also calculates the cost for 5 hours, which seems to be the evidence to support the claim that Music Makers is "more cost effective if Prom is longer than 4 hours." The student claims Dance Partners costs less if prom is less than 4 hours. No evidence is provided for this claim.

The warrants are not explicitly stated. The warrant for the costs being equal at four hours is: if both fee structures are modeled properly, and the two costs set equal, then solving finds the number of hours for which the costs are the same. The warrant that Music Makers is cheaper after 4 hours rests on knowing that the equations are linear. Linear equations do not "turn around," and so if MM is cheaper for one value (here,  $x=5$ ) that is greater than the number of hours at the point of intersection ( $x=4$ ), then it is cheaper for all values of hours greater than the point of intersection.

The argument could be strengthened by labeling the equations (which equation represents which company?), and by providing a reason for why the equations were set equal and why Dance Partners is cheaper before 4 hours. The argument could also be strengthened by explaining how the equations  $200 + 175x = C$  and  $225x = C$  represent the cost, but whether this needs to be included depends on the class and students' prior background.

# Student 1

1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

Music Makers : \$200 + \$175 per hour

2 hours = \$550

3 hours = \$750

4 hours = \$900

6 hours = \$1,250

8 hours = \$1,600

Dance Partners : \$225 per hour

2 hours = \$450

3 hours = \$675

4 hours = \$900

6 hours = \$1,350

8 hours = \$1,800

Depending on how many hours you would have the dj for would effect your decision. If you were to have the dj for only 2-3 hours, Dance Partners would be the cheaper choice, if you wanted the dj for 4+ hours, Music Makers would be the best choice.

## Commentary

This student's argument was categorized as **ADEQUATE quality**.

The student claims that it depends on the length of the prom. The student provides evidence of multiple cases, including showing the number of hours for which the two companies cost the same. The student does not state the companies cost the same at 4 hours even though the evidence shows that, instead claiming that MM is the best choice for 4 hours (and beyond). The warrant that supports the claim is directly comparing the costs by the addition of the initial fees and costs per hour.

The argument could be strengthened by stating that either company is cost effective at 4 hours. It could also be strengthened by either making explicit the assumption that a prom lasts at least two hours, or adjusting the claim to state that Dance Partners is better for any prom shorter than 4 hours.

Note that the Music Makers costs for 3 hours is incorrect, but this does not detract from the overall argument.



# Student 4

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

$$\begin{array}{r} \text{DJ A} \\ \text{Fee - } 200\$ \\ + \\ 175 \times \text{hours} \end{array}$$

$$\begin{array}{r} 1 \text{ hour} = 375 \\ 2 \text{ hour} = 550 \\ 3 \text{ hour} = 725 \\ 4 \text{ hours} = 900 \\ 5 \text{ hours} = 1100 \end{array}$$

$$\begin{array}{r} \text{DJ B} \\ \text{Fee - } 0\$ \\ 225 \text{ hour} \end{array}$$

$$\begin{array}{r} 1 \text{ hour} = 225 \\ 2 \text{ hour} = 450 \\ 3 \text{ hours} = 675 \\ 4 \text{ hours} = 900 \\ 5 \text{ hours} = 1125 \end{array}$$

Dance Partners is most effective because after 3 hours of DJ the cost was only 675.00\$ unlike music makers who charged 750.00\$ until 3 hours of DJ when the cost was 900.00\$ and dance partners was 1125.00\$

## Commentary

This student's argument was categorized as **LOW quality**.

The student has two claims: Dance Partners (DJ B) is more cost effective and Music Makers costs more until 5 hours. To support the claim that Dance Partners (DJ B) is more cost effective, the student offers an example of 3 hours and compares the costs. Similarly, to support the claim that Music Makers always costs more until 5 hours (when presumably that changes), the student compares the costs in the table, identifying the cost of each for 5 hours. Note: The evidence that supports these two claims is mathematically incorrect although the student is making the proper inferences (connecting evidence to claims). The warrant is implied (directly comparing costs by adding costs for each hour).

To strengthen the argument, the student should more clearly state the claim(s), identify who DJ A and DJ B are, and fix mathematical errors. The student should also consider what evidence helps to support the claims made.

# Student 9

1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$ 175per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

$$\begin{array}{l} \text{Music Makers: } 175x + 200 \\ \text{Dance partners: } 225x \end{array} \quad (\text{plug in})$$

Answer: Music Makers will be more cost effective if you rent this company for over 4 hours. But if you rent the group for only four hours, they both will be good because they both cost \$900 at 4 hours. But if you rent it for less than 4 hours, Dance Partners would be better.

## Commentary

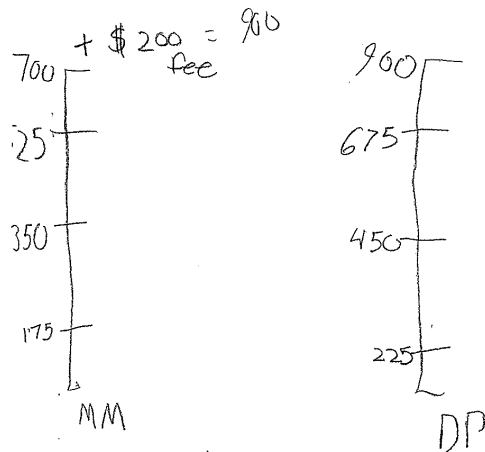
This student's argument was categorized as **LOW quality**.

The student claims that Music Makers will be more cost effective if you rent for more than 4 hours, both companies will be cost effective at 4 hours, and Dance Partners will be more cost effective before 4 hours. The evidence provided is the two equations and noting that both companies charge \$900 for 4 hours. No warrant is provided that connects the evidence offered with the claims.

To strengthen the argument, the student would need to provide additional evidence, such as showing how the \$900 was derived or offering some explanation for why those equations model the problem situation. The student does write "plug in" which may hint to how the \$900 was derived, but this is not clear. The student should also provide evidence and reasoning to support the claim that Music Makers is more cost effective for proms longer than 4 hours and Dance Partners is more cost effective for proms shorter than 4 hours.

# Student 6

1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.



Both music companies would charge you the same price for prom

## Commentary

This student's argument was categorized as **LOW quality**.

The student does not make an explicit claim about which company is more cost effective, but the student makes a closely related statement, "both companies would charge you the same price for prom," and so presumably are both equally cost effective. This claim is only true if the prom is 4 hours. It could be that the student is assuming prom is 4 hours and therefore suggesting both companies are the same price. However, the assumption is not made clear.

The evidence is a chart of costs which shows the amount attributed to the hourly rates for each company, and then adds the \$200 fee to the 4 hour-cost for Music Makers.

The warrant is the direct comparison of the calculated costs.

The argument could be strengthened by explicitly stating that only a 4-hour prom is being considered or by considering all reasonable cases.

The argument could be further strengthened by having the chart of costs explicitly labeled as this is not a standard representation and is left for interpretation.

# Student 7

- 1.) In preparation for the Prom, students are researching the costs of two local DJ companies. Music Makers charges a fee of \$200 and an additional \$175 per hour. Dance Partners does not charge an initial fee, but charges \$225 per hour. Which company would be more cost effective for the prom committee? Write a mathematical argument to support your decision.

Music Makers:  $\begin{matrix} \text{start} \\ \$200 \end{matrix} + \begin{matrix} \text{Per hour} \\ \$175 \end{matrix}$

Dance partners:  $\begin{matrix} \text{start} \\ \text{initial} \end{matrix} + \begin{matrix} \text{Per hour} \\ \$225 \end{matrix}$

$$x=5 \quad 200 + 175x = 1075$$

$$225x = 1125$$

I think that Dance partners DJ company would be more cost effective. I think this because no matter the amount of hours(x) it will still cost more than Music Makers.

## Commentary

This student's argument was categorized as **LOW quality**.

The student claims that Dance Partners is more cost effective, and supports this by saying "no matter the amount of hours," Dance Partners will cost more. Notice the contradiction between the student saying Dance Partners is "more cost effective" and Dance Partners will "cost more than Music Makers." There may be some lack of understanding of the phrase "more cost effective."

The evidence offered (although not connected to the claim) is two equations for the fee structures of both companies (both are correct) and the cost of each company at 5 hours (also correct). No warrant is explicitly stated. For the student's claim to follow from this evidence, the warrant would be: if a company costs more for 5 hours, it costs more for any number of hours. This is a faulty inference. In addition, in this particular problem, the argument cannot be based only on the value at one point (i.e.:  $x=5$ ).

The argument could be strengthened by having the equations labeled, having the claim follow from the evidence rather than suggesting that one point can tell the whole story. Alternately, the student could specify that a typical prom lasts 5 hours and use that evidence to support the claim for this typical prom.

# Key Connecting Sorting Packet to Argumentation Resource Packet

Student number (Sorting Packet)	Resource Packet Sample
1	Adequate
2	High
3	Adequate
4	Low
5	High
6	Low
7	Low
8	Adequate
9	Low
10	High

Resource Packet Samples (Quality)	Student number (Sorting Packet)
High	2
High	10
High	5
Adequate	3
Adequate	8
Adequate	1
Low	4
Low	9
Low	6
Low	7