Facilitation Guide

Module 3: Focus on Implementation – Norms and Routines to Prompt and Support Argumentation

This module is one of five modules created for professional learning purposes as part of the Bridging Math Practices project. An Overview for our facilitation guides and the modules is available at http://bridges.education.uconn.edu/argumentation-pd-modules/. This module can be used independently or in conjunction with one or more of the other four modules. We encourage users to become familiar with the set of materials and then adapt them to their particular needs and timeframe.

This Facilitation Guide includes the following:

- Goals for Module 3
- Background Information on norms and routines
- List of Materials Needed for Module 3
- Timing Table for Module 3 Activities
- Implementation Guide and Possibilities
    - Detailed description of each activity and suggestions for implementation
- References
- Additional Resources

All handouts and other materials for Module 3 can be found at http://bridges.education.uconn.edu/norms-and-routines/

Goals: Module 3

In this module, participants will

- Develop a deeper understanding of argumentation and its potential in the classroom
- Examine norms and routines that can support mathematical argumentation in the classroom
- Develop an understanding of a pedagogy of inquiry to support mathematical argumentation in the classroom

Background Information:

This module focuses on *norms* and *routines* that can help prompt and support mathematical argumentation in the classroom. It builds on Modules 1 and 2, as we draw on our knowledge of argumentation and mathematics tasks, to think in more detailed ways about the classroom and how to make argumentation happen.

We focus on *norms* and *routines* in this module about implementing argumentation in the classroom.

- *Classroom norms* are critical because they influence all interactions. They exist whether we are aware of them or not. They can be supportive of argumentation or undermine efforts and require explicit attention.
- *Classroom routines* are "sequences of actions regularly followed" that provide structure for a lesson. A routine's structure is critical for organizing the intellectual and social work of argumentation. They allow for the emergence of many ideas, rich conversations, and help maintain a focus on learning goals.

In Module 4, we continue the focus on implementation, zooming in closer to examine classroom interactions and patterns of questioning in mathematical discourse.

Materials:

Copies of handouts
Slides to project
Technology to play a web-based video, with audio
Talk Frame Icons (1 set for PLC format; suggested multiple sets for Workshop format)
Hexagon and Square tiles (recommended)
Manipulatives for the Bridging-to-Practice Activity, as appropriate

Workflow Table for Module 3

Session activity and focus	Estimated Timing		Materials
	Monthly (1.5 hrs)	Workshop (3.5 hrs)	
Opening Activities: Monthly PLC: Participants share their "Between Sessions" work Workshop: Community or Problem Solving	5-10 mins	(as appropriate for workshop timing)	Completed Opening Activities Template
Activity 3.1 Norms and Establishing a Culture of Inquiry Participants **brainstorm** norms that support argumentation, **view a video** of a teacher working to establishing norms of argumentation, **analyze the video**, and reflect again on the norms and their own practice	25-30 mins	40 mins	Capacity to play video, with audio Handout 1: *Brainstorm Norms* Handout 2: *Class Background* Handout 3: *Video Viewing Questions* Handout 4: *Video Clip Transcript* Handout 5: *Additional Material on Norms* (optional)
Activity 3.2 Pedagogical Routines that Support a Pedagogy of Inquiry: Talk Frame 3.2.1 Overview and Introduction 3.2.2 Talk Frame activity 3.2.3 Debrief	50 mins .1 (10) .2 (30) .3 (10)	70 mins .1 (10) .2 (40) .3 (15)	Handout 6: *Pedagogy Routines Brainstorm* Handout 7: *Pedagogical Model Support a Culture of Thinking* Handout 7: *Chain of Flowers Pattern Task* Handout 9: *Debriefing the Talk Frame Routine* Handout 10: *Talk Frame Overview* Handout 11: *Talk Frame – Planning Template and Examples* One set of Talk Frame icons (or substitute) Tile manipulatives (optional)
Activity 3.3 Examining Additional Talk Frame Examples	n/a	20 mins	Handout (optional): *Mod3 Addtl Resources – ATOMIC 2014* (secondary tasks)
Activity 3.4 Bridging to Practice: Monthly PLC Format: Explain work to be completed between sessions Workshop Format: Team-teach mini lessons using the Talk Frame Routine	3 mins	70 mins	Handout: *3Bridging_Mini Lesson Tasks* Handout: *3Bridging_Mini Talk Frame Lesson* Manipulatives, as appropriate (optional)
Activity 3.5 Session Closure Reflect on day's session, synthesize key points, and/or administer a feedback survey	2 mins	10 mins	Handout 12: *Reflecting on Norms*

Implementation Guide and Possibilities: Module 3

In the sections that follow we provide suggestions on how to use the materials for two different models of professional development: monthly meetings during the school year and an intensive five-day workshop. We also include the goals of specific activities (indicating how they contribute to the goals of the module) and some of our reasoning for including particular activities and/or materials. Following each activity description, we include a table with common issues for the different activities and suggest questions or prompts you might use to help address those issues.

Opening Activities

Monthly PLC Format

In the monthly PLC, you might organize participants into pairs or groups of three to debrief their Bridging-to-Practice work from Module 2 related to tasks. For example, participants may have selected or adapted a task that they intend to support student argumentation in mathematics, and implemented it. Participants could bring copies of their task and may have copies of student work as well. We have found that participants often find that seeing the tasks used by the colleagues can be informative. You might consider making the tasks participants bring available to the group.

Workshop Format

In the workshop format, you might use this time to engage participants in doing mathematics. As always, choose a problem you think will work for your particular group. We chose the illustrativemathematics problem, Animal Populations (see handout, and also available at https://www.illustrativemathematics.org/content-standards/tasks/436).

There are many different approaches to the Animal Populations problem, some of which are outlined on the illustrativemathematics website. Note that the core question for this problem is *how are you going to compare?* No matter the argument produced, teachers and students must find a valid way to compare two quantities and be able to explain how the comparison was made and the result of that comparison. With respect to argumentation, this problem can provide opportunities to talk about how to move from testing values or cases (which helps you develop a sense of how things "work") to more general claims and more general arguments.

The problem may also cause people to feel a little overwhelmed at first: What are these symbols? How am I ever going to attack this? This element could be worth discussion as well – linking it to how we could help students manage these moments.

Another math problem that can be posed is the problem posed in the video later in this session:

Which is larger, 6/10 or 4/6?

Participants can produce arguments to support their claims and examine the many different approaches to this problem. As with the Animal Population problem, the core of this problem is finding a way to appropriately compare the quantities.

Another option for this time is to revisit the Community Agreements. You might ask participants what they think the group is doing well, and which agreements they might want to give some extra attention to today.

Module Objectives

Prior to Activity 3.1, the module objectives should be shared.

Participants will

- Develop a deeper understanding of argumentation and its potential in the classroom
- Examine norms and routines that can support mathematical argumentation in the classroom
- Develop an understanding of a pedagogy of inquiry to support mathematical argumentation in the classroom

Activity 3.1: Norms and Establishing a Culture of inquiry

The purpose of this activity is to provide participants with the opportunity to reflect on and extend their ideas about the norms of a classroom that are supportive of a *culture of inquiry* and *argumentation*. One cannot have a classroom where students are interested in one another's ideas, and willing to develop and question their own and other's ideas, unless there is a *culture of inquiry* to guide the class's work together regardless of the particulars of the activity.

As a result of this activity, participants can be more deliberate in thinking about their own classroom norms (what exists, what norms might need to be developed). In particular, they can be more deliberate in thinking about similarities and differences between classrooms that support "math talk" or lots of participation versus those that support participation in mathematical argumentation and a vibrant thinking culture.

Brainstorm

This activity starts with a brainstorm of the norms that are needed to support student argumentation, which we couch in this broader context of supporting a culture of inquiry. Handout 1: *Brainstorm Norms* provides a space for participants to record their ideas.

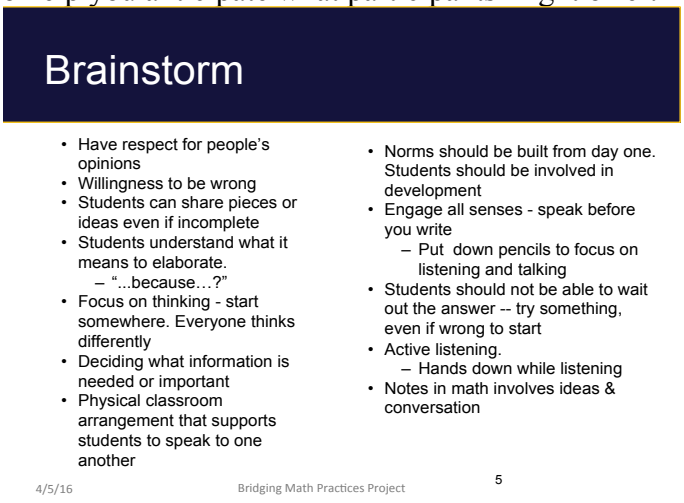
You might have participants do a think-pair-share, or individually record ideas and then develop a collective list to share with the group. We suggest that you record participant ideas on the powerpoint slide as they share them [a titled-but-otherwise-blank slide has been included for your use], or record in some alternate format (e.g., chart paper).

You may choose to let the list stand as a "brainstorm," or you may choose to discuss it further (at that time, or revisit later) to refine the list or choose a "top 3" that participants feel are needed to have a classroom culture for students to participate in mathematical argumentation. The list can also provide a reference point for discussing the video clip, and potentially extending the list.

Potential points to be raised or issues	Possible questions or prompts
Norms are often implicit, so even if teachers think they are establishing particular expectations, students might not be aware or have the same understanding	How do you know if a norm is in place in a particular classroom? How could we help students to follow this norm or

as the teacher	expectation?
Teachers may notice that the norms in their classroom are already supportive of mathematical argumentation	You may simply acknowledge this. You might also leverage this person's expertise, asking more about which norms they find most challenging to develop (and how they do it) or why they developed these norms (if not to support argumentation) and whether any adjustments might be needed if they engage students more extensively with argumentation.
Classrooms that support "math talk" or lots of participation may be different than those that support participation in mathematical argumentation.	One key difference is that in a classroom engaged in argumentation, students must *attend to other students' ideas.* This feature may require a big shift for students who are used to listening only to authoritative voices on a topic (e.g., the teacher), and may not initially find value in listening to and making sense of a peer's idea. A follow up discussion at some point regarding how to help students value listening to one another could be productive. (Handout 5 also includes some pointers on this.)
Participants contribute narrow expectations focused on specific skills. For example "students must know their math facts" or "students use good vocabulary to express their ideas"	How does knowing math facts [using good vocabulary] support argumentation? In what ways could this expectation constrain participation from some students? Encourage participants to focus on the culture of the classroom at this time and the ways students interact. Students at all levels can engage argumentation – it may look different, and there may be different things to sort out and argue about, but mastery of vocabulary or math facts is not a pre-condition for engaging argumentation.

Here is an example set of norms brainstormed by one of our cohorts of secondary participants for reference and to help you anticipate what participants might offer:



Norms "In Action"

In this part of the sequence, participants watch and discuss a video clip from one teacher's lesson. This short clip offers insight into how one teacher, in one setting, at one point in time, worked to instantiate and further establish norms to support student participation in argumentation.

For the Workshop Format, have participants work through **the math task** first. You might also elect to do this with the PLC format if you feel doing the problem is needed to help participants "get into" and make sense of the video clip. This problem also could be part of the Opening Activities at the start of the session.

Math Task Prompt:

Which is larger, 6/10 or 4/6?

With time, this work can be followed by discussion of how students might approach the problem, including what participants expect students might find challenging, and key ideas that could be discussed in the classroom (which relates to the goals for the task). Note that this task, as implemented in the video, was done using a Talk Frame routine, which is a routine to support argumentation and the topic of the next activity in Module 3.

Prior to showing the video clip, be sure to orient participants and **establish some guidelines for how to view classroom videos**. Doing so is critical to having a subsequent productive conversation, as you want participants to focus on key questions and not make claims about the classroom and teaching for which there is not enough information to render an informed comment or judgment. You might do this work through questioning, or you might choose to share the draft slides included in the powerpoint and outline your perspective on the value. There is also a 1:46-minute set of narrated powerpoint slides about how to productively watch video that you could show, available at https://youtu.be/dCs8dxhzx6c.

The Video: Michelle McKnight's Sixth-Grade Intervention Class

To set up the video, provide some context (e.g., 6th-grade intervention students in a pull-out support setting) and read the guiding questions participants should consider while viewing the video. You could read together, or summarize for the group, the one-page handout the Michelle contributed about the class and context, Handout 2: *Class Background*. Many also find it help to have the transcript to mark notable moves and to refer to later during discussion. A transcript is included with the module materials (Handout 4: *Video Clip Transcript-Norms*). The video itself also has subtitles. The video clip is 3:47 minutes long. Handout 3: *Video Viewing Questions* includes the following guiding questions:

1. What do you see in this video that relates to culture of thinking and argumentation?
2. What moves do you see the teacher making to help promote a culture of thinking and argumentation?
3. What norms may have been previously established in this class to support this interaction? What evidence does the video provide?

Watch the video (3:47 mins). https://youtu.be/I1nzLeGpdDc

There are many ways to organize the subsequent discussion – in pairs, small groups, whole group; by question; starting with "reactions," etc. We suggest you consider the group's needs and goals, and also strongly encourage you to provide more than only whole group discussion. With the whole group, as a facilitator, you sometimes hear only one slice of ideas and "skeptics" (of a prevailing idea) or those with questions do not feel there is space to raise them.

Here are some ideas in response to the Guiding Questions:

1. What do you see in this video that relates to culture of thinking and argumentation? [Note: some bullets listed for #2 below are specific moves that support observations listed here in #1]	• Students are sharing their ideas • Students are working on sharing their arguments and not just steps of what they did • Students are at least respectful of one another (we don't have evidence whether they're really attending to each other's ideas) • The teacher does not step in as the authority to explain the student's idea, or to be the first judge of the student's idea • By not addressing the "incorrect" answer right away, the teacher has created an opportunity for students to keep thinking and eventually critique this argument
2. What moves do you see the teacher making to help promote a culture of thinking and argumentation?	• Has a student turn his body to face peers – indicating they are his audience and he has something to say • "Remember we're going to try to stay away from telling step by step what you did. We want to talk about *why* you did that …" • "How does what you did compare 6/10 to 4/6?" *Note the importance of this kind of question. It focuses students on the core idea of this problem – how are you making a comparison.* • Revoicing students' contributions: e.g., "you're saying…" • The teacher seemed to pick up on a mismatch between the student's written assertion of which was larger and his verbal assertion of which was larger and, after listening more to his idea, helped him correct his use of the symbolic notation
3. What norms may have been previously established in this class to support this interaction? What evidence does the video provide?	*Note that responding to this question requires a little more inference, as we have 4 minutes of information. These are possible norms participants might note, with some evidence.* • Students are respectful of a presenter (evidence: no student called out, interrupted, or put another student down) • Students are thinkers, and math is about thinking (evidence: the student seem to be sharing *his* idea and not the "right" approach; multiple approaches are put on the board; students seem used to explaining their ideas) • Everyone has something to share (evidence: there are three student's work on the board (of 6) and students seem to be familiar with sharing their ideas) • There is more than one way to approach a problem

Brief discussion of the math of the task and "incorrect" answer:
This video ends with a student having made an argument that is *not* viable, although the student has asserted the right answer. It is true that 4/6 is larger than 6/10, but it is not the case that 4/6 is larger than 6/10 for the reason that "4 is closer to the denominator." (Using our language from Module 1, we see the student's claim is correct, but the warrant is faulty.) This idea that "closer numbers make a larger fraction" is a common error among students, and has some reasonable mathematics behind it, as do most errors. It is the case that, when the differential between the value of the numerator and the value of the denominator is small, the fraction is quite "large," meaning close to one. For example, consider each pair, noticing that when the numerator is "closer to" the denominator, the fraction is larger.

4/6 and 3/6 ["4 is closer to 6 than 3 is to 6."]
8/10 and 6/10 ["8 is closer to 10 than 6 is to 10"]

Consequently, if your denominator (number of parts) is constant, the idea that "closer numbers make a larger fraction" does hold true. The argument to support this can be stated: when you have a fixed denominator, "closer numbers" means there are more parts of the whole, and so the fraction that has the numerator is closer to the denominator will be the larger fraction. (Note: we are assuming here for ease that we are talking about proper fractions and not improper fractions.)

Looking at this difference, however, is not a mathematically sound approach in general for comparing the size of pairs of fractions. One would reach the wrong conclusion using this "rule" when comparing the following pairs:

4/6 and 7/10 (4/6 is larger because 4 is closer to 6 than 7 is to 10)
4/6 and 1/3 (the assertion would be that the fractions are equally large)

The wrong conclusion is reached because the number of parts of the whole *alone* does not make the magnitude of the fraction. The *size* of the parts is critically important as well. This mathematical issue would need to be addressed at some point for students to develop their understanding of fractions and ways to compare fractions.

Additional discussion and questions:
As video is so rich and powerful, we expect there will be additional comments and ideas prompted by this video. The following are some questions and concerns offered by our cohorts of participants that indicate potential topics of interest to participants in your group.

- In this video, the teacher seems to be strongly involved in the questioning and directing the lesson. Does the teacher "release" this responsibility over time? What does this look like? How do we teach students to take on more responsibility? [Implicit in this is a question about whether one *should* and *can* turn this more over to students.]
- The video ends with an incorrect assertion. How long can a teacher let a wrong idea go unaddressed? Is it OK for the goal to be discourse and student engagement, and not worry about the right answer? Will students be confused if a wrong answer is left up on the board?

We also find this video useful for raising some larger points:

- All students can engage in argumentation, as all people can reason. The tools they use will vary, and the nature of the work will vary, but all students can engage and get better at this powerful mathematical practice.
- Supporting argumentation requires deliberate attention to *ideas* and the *decisions and choices students make.* It further requires moving beyond recounting steps and showing computations. Steps and computations are how one works out one's ideas to find the results, but the approach is the key aspect.
- Argumentation is extremely valuable for formative assessment purposes. Particularly in an intervention class, where students may have uneven "gaps" in prior expected knowledge, argumentation helps teachers learn more about what their students know and where they need more work.
- Argumentation is language intensive and centralizes the *communication* of ideas. Alongside developing argumentation skills, teachers must attend to language and how mathematical ideas are being represented.

At the conclusion of this segment of Module 3, we encourage you to have participants synthesize some of their learning or new ideas from this video and reflective discussion. In the set of handouts, there is also 4-page handout that offers some additional ideas and strategies for supporting norms for a culture of inquiry that you may wish to share or discuss with participants. At this point, we turn to *classroom pedagogical routines* to support argumentation.

Activity 3.2 Pedagogical Routines that Support a Culture of Inquiry: Talk Frame

This activity introduces and engaged participants in one particular pedagogical routine to support argumentation and a culture of inquire. We have organized the larger activity into three parts to make the facilitation guide easier to follow: 3.2.1 Overview and Introduction, 3.2.2 Talk Frame activity, and 3.2.3 Debrief.

3.2.1 Overview and Introduction

Similar to how norms are foundational for supporting a culture of inquiry, pedagogical routines can serve as important tools to organize the work in mathematics classrooms in ways that support inquiry and argumentation. We define a routine as "a sequence of actions regularly followed."

To help participants transition to thinking about routines, you might share this definition and ask participants to brainstorm, "What routines do you know that help support argumentation and a culture of inquiry?" Handout 6: *Pedagogy Routines Brainstorm* and a titled powerpoint slide have been included to record individual and group ideas.

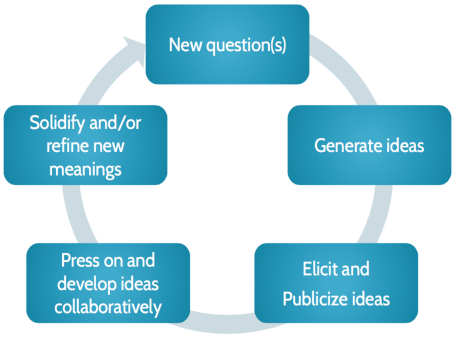
After participants have time to think, you may choose to have some participants share out their ideas. Responses to this question could include:

- Think-pair-share
- Number Talks
- Launch-Explore-Summarize (a format used by Connected Math Project)

This conversation does not need to be lengthy. The primary purpose is to help participants think about what a routine is and how they are likely already familiar with some routines.

A Pedagogical Model to Support a Culture of Thinking

We found it useful to share the figure to the right as a visual model for pedagogical routines that support argumentation and inquiry. This figure is available in the handouts and powerpoint slide. When sharing this figure, focus participants' attention to the cyclical nature of this diagram. As new ideas and questions are shared and publicized, additional attention is given to those ideas to press and collaboratively develop meaning. It is also important to note that this model moves beyond having students *share* ideas. A good first step toward a pedagogy of inquiry is to have students share ideas. A necessary, and more challenging, next step is for students to work on ideas together as they listen to one another and discuss mathematics together. Discussions lead the class can work to solidify what they learned and use those new ideas moving forward to their next questions.



Any routine that supports students to engage in the cycle depicted in the figure can offer opportunities for students to develop *a culture of inquiry*. Additional attention to reasoning may be necessary to move that one step further and support the *practice of mathematical argumentation.*

3.2.2 Talk Frame activity

In this portion of the activity, participants engage in a specific pedagogical routine, a Talk Frame, considering the routine both from the perspective of students as well as from their own perspective as teachers/coaches/etc.

To set up for the Talk Frame activity, we encourage you to discuss with participants: (a) why a group of (presumably good-at-math) math teachers might do problem solving together, and (b) the norms the group has established to govern its interactions. Participants can feel a bit "on the spot" or nervous about doing math in front other professionals, depending on their prior experiences and relationships with others in the room. This could be particularly true if you are doing this work in the monthly PLC format and the prior sessions have not provided many opportunities for participants to do math together and/or if you are working across grade levels. Explicitly discussing the purposes and revisiting the norms can help participants feel more comfortable and think more carefully about their interactions with others as the process unfolds.

Here are some points you might share regarding why mathematics teachers can and should do math problems together:

- Provides a common math-teaching-and-learning experience across the group for reflection and discussion
- Gives opportunities to discuss math tasks and task implementation
- Can expose participants to particular teaching strategies

- Offers opportunities to see how others think
- Provides an opportunity to engage in a productive struggle in much the same way we would like our students to do
- Can be fun and help us learn more math and see more connections

Overview of the Talk Frame Routine:
We strongly encourage you to read through Handout 3.10 *Talk Frame Overview,* as this provides a useful overview of the Talk Frame routine. This document was prepared by Tutita Casa, one of the authors and leaders of Project M^2 that developed this routine. Please also reference the powerpoint slides and other materials to gain a sense of this routine. This routine is quite similar to the 5-Practice Routine (Smith & Stein, 2011) but the Talk Frame does not include explicit phases related to planning (though teachers should absolutely do this preparation work).

Broadly, the Talk Frame routine begins with the launch or "Think" phase. It is here that the teacher poses the math task or question. Next, students/participants are given sufficient time to work individually (and possibly in groups depending on your goals) to generate one or more solutions to the problem. During this work time, the teacher identifies a few complementary "Talk Ideas" to be shared and discussed as a class.

The selection of these "Talk Ideas" involves important pedagogical decisions about what mathematical ideas the teacher elects to foreground and which students s/he selects to participate. This phase of the routine may prove to be a rich point of discussion with participants later. Finally, the routine concludes with the group establishing one or more statements about what "We Understand" related to the question or task that was posed. The "We Understand" is one of the most important aspects of this routine because it provides an opportunity for the teacher to summarize, solidify, and refine the new understandings that students generated during the "Think" and "Talk" phases.

The Chain of Flower Pattern Task
We recommend that during the implementation of this task that you serve as the teacher and your participants are fully and authentically in the student role. There is an opportunity after to debrief the structure and how the lesson unfolded, and allow participants to put on their "teacher hat." To begin the activity, you might provide a brief overview of the structure to give students/participants a sense of the routine, but include details as the lesson unfolds. That is, we do not suggest going through the overview of the Talk Frame in detail first. Rather, have participants experience the routine as students first.

Think: Chain of Flower Pattern Task
We elected to pose a non-standard pattern task as the "Think" question in these materials. The problem is below and on Handout 8: *Chain of Flowers Pattern Task*. You could use any number of pattern tasks (or other tasks) effectively. This flower pattern is akin to one idea presented in Friel & Markworth (2009); you may wish to look at the article for ideas for other patterns to analyze.

Consider the following pattern:

Figure 1

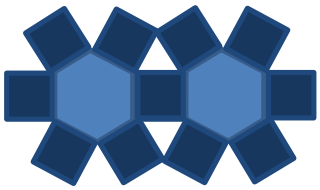


Figure 2

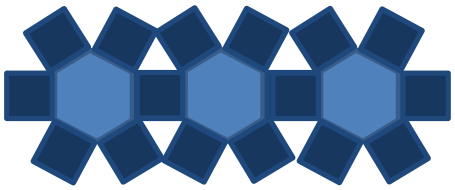


Figure 3

a) Draw Figure 5. How many tiles does it have?
b) How many tiles will the 25th figure have? How do you know?
c) How many tiles are in the *n*th figure? How do you know?

Individual work: Participants will likely need 5-10 minutes to work on the problem individually.

Small group discussions: Ask participants to share their ideas in their small groups. The Talk Frame does not require this component, but it is useful to allow participants to share their ideas, further develop their ideas, and for you to learn more about their thinking.

Select talk ideas: During this time, circulate to identify which participants you will ask to share solutions as talk ideas. We suggest that you look for three ideas. (Two or four can be fine too.)

When selecting talk ideas, consider both the mathematical goals you have for the lesson, as well as any social goals you have related to individuals in your group. For example, is there a participant or student who has not talked much during whole group discussion who has a novel approach to the problem? This could be a good opportunity to highlight the strengths or competence of that particular participant/student.

Below are few possible solution strategies that we have seen participants and students use to solve this task. You can use these anticipated strategies to help you decide what you might look for in the talk ideas you select.

First, Figure 5 will look like:
There are 5 hexagon tiles and 26 square tiles. This gives a total of 31 tiles for the whole figure.

We include here some potential solutions for determining the number of tiles in the n*th* figure.

Solution A: Analyzed how the figure grows – Seeing "plus 6"

Figure #	1	2	3	4	5	…	25	…	nth Figure
# Tiles	7	7+6 = 13	7+6+6= 19	7+6+6+6= 25	7+6(4)= 31		7+6(24)=151		7+6(n−1) = 7+6n−6 = 6n + 1

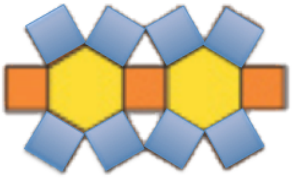
Each figure adds one hexagon and five orange squares, for a total of 6 additional titles. Therefore, the 25th figure will include the original 7 titles from the first picture, plus 24 copies of the pattern each contributing an additional 6 tiles.

Based on my explanation from part b above, I know that each new figure contributes 6 additional titles to the picture. In the first figure, however, includes 7 total tiles. This discrepancy is due to an orange tile around the outside that gets double counted as I attach new hexagon patterns onto the train in later figures. The + 1 on the end of my expression is tied to that square tile.

$7+6(n-1) = 7+6n-6 = 6n+1$

Solution B: Analyzing the figure – Seeing "Top/Bottom" and "Side" tiles

As the hexagons are added onto the train for each figure, you can picture the square titles as two sets: (a) the four titles that make up the top and bottom – highlighted in blue, and (b) the side tiles –highlighted in orange.



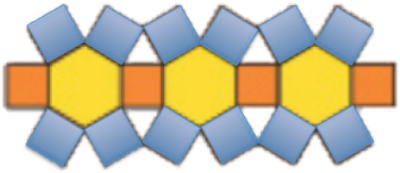


Figure 1: 4 top/bottom + 2 sides + 1 yellow hexagon = 7 tiles

Figure 2: Adds on 4 more top/bottom + 1 side + 1 yellow hexagon = 7 + 4 + 1 + 1 = 13

Figure 3: Adds on 4 more top/bottom + 1 side + 1 yellow = 13 + 4 + 1 + 1 = 19

Figure 5: Adds to Fig 3 two more sets of 4 top/bottom + 2 more sides + 2 more yellow = 19+8+2+2 =31

Figure 25: Figure 1 + 24 copies of (4 top/bottom + 1 side + 1 yellow) = 7 + 24(6) = 151

Figure n: Figure 1 + (n–1) copies of (4 top/bottom + 1 side + 1 yellow) = 7 + (n–1)(6) = 6n + 1

Solution C: Reasoning Proportionally - and Miscounted Overlap

Figure 1 has a center yellow hexagon with a square tile on each of its six sides. This figure has a total of 7 tiles.

Since I know that each center yellow tile is a hexagon (with 6 sides). I know that each new hexagon adds on another 7 tiles total. That is, 1 more center yellow plus 6 more squares.

So, Figure 5 has 5(7) = 35 tiles
Figure 25 has 25(7) = 175 tiles
Figure n has 7n tiles.

Solution D: Using Number Patterns

Figure 1 has 7 tiles; figure 2 has 13 tiles; figure 3 has 19 tiles. So I see the values are increasing by 6 each time. It's not just figure number times 6, however, as you have to add 1 more to make the numbers work. 6* Figure number + 1.

Sharing "Talk Ideas:" At this point you should have participants share out the specific Talk Ideas you selected while they were working. Participants should share these publicly. You may wish to have them write up their ideas while small groups are still talking. The Talk Frame Icons – specifically the ones labeled Talk Idea – can be used to help organize the board/public space. Importantly, however you decide to make these Talk Ideas public, participants must have the opportunity to explain the *reasoning* behind their work. You may need to provide explicit prompting, such as, "Can you tell us how you know that is true?" or "Please explain why/how you included that [expression, term, etc.]?"

Which ideas should be shared?

The Talk Ideas shared should be selected in relation to your goal or purpose. There are many different goals you might pursue with this task, and the goal may in part be shaped by the work you see. Here are some possible goals for this task. (And you may wish to plan your implementation using the Lenses from Module 2.)

- Connect across representations: the goal may be to explore the "plus 6" and how it is represented different ways depending on whether you use a graph, equation, table of values, etc., as well as understanding where the "plus 6" is in the diagram.
- Structure: the goal may be to examine how different expressions reflects how participants *see* the flower chains, or how they are visualizing the pattern. You might choose to look carefully at the expressions generated for the number of tiles and see how those reflect these different ways of seeing.
- Proportionality: the goal may be to the relationship between figure number and number of tiles and determine whether it is a proportional relationship or not, and how we know.
- Argumentation: the goal is to compare and analyze the arguments offered in the Talk Ideas and have students/participants reflect on the strengths of each, and areas where each might be revised to make the argument stronger. In particular, it is worth discussing that finding a number pattern is valuable, but leaves open the question of whether the pattern will continue, and if it does, how we know and what is driving that consistent pattern. (Similarly, for those who see it "grows by 6" and conclude "so the slope is 6" and write a linear equation, a question might be: how do we know that the pattern is linear?)

We Understand: It is here in this phase that you might press on particular ideas in order to highlight certain mathematical goals. This discussion can be open-ended to start with, perhaps posing a general question about what they learned and allowing many participants to contribute ideas. Then you can work to streamline the set of ideas generated to just one or two main concepts that you want to highlight. The "We Understand" piece will connect with the goal you pursued in the discussion of the Talk Ideas.

3.2.3 Debrief of Talk Frame Activity

To close Activity 3.2, we return the focus to the pedagogical aspects of the lesson. Handout 9: *Debriefing the Talk Frame Routine* can be helpful for supporting the discussion. A first goal is to have participants unpack their own experience with the Talk Frame and start to consider how the structure/routine of the Talk Frame supported their engagement, as well as how particular teacher (facilitator) moves or decisions may have shaped the lessons.

This discussion might then turn to a review the Talk Frame Routine structure. Handout 10: *Talk Frame Overview* offers a brief explanation of each component in the routine. You might also reflect on some of the specific decisions and choices you made while you were implementing the routine, or ask participants about the questions they have about the implementation and potential choices you made.

Here are some questions (some on the handout as well as some additional questions) that can be used for with small groups or the whole group as well:

- What could the Talk Frame, or a similar routine, help you do?
- What questions do you have?
- How does the Talk Frame allow students to generate ideas?
- How can the teacher facilitate this process?
- How does the teacher facilitate this process in order to promote thinking?
- What does the teacher do to help students make connections between ideas and solidify their learning?

As this activity is brought to a close, you might pose the final question:
How does the Talk Frame Routine support the Pedagogical Model for a Culture of Thinking?
Have participants share their ideas.

Handout 11: *Talk Frame Planning Template and Examples* can be shared at this point. The handout includes both a template to support planning and two completed examples, one for the Chain of Flowers task and one for the question: Without using the traditional algorithm, can you make sense of 1 ÷ 2/3? For the PLC format, this handout will primarily be for future reference. For the Workshop model format, this handout supports the next activity and further discussion of the Talk Frame.

Activity 3.3 Examining Additional Talk Frame Examples (*Workshop Model only*)

Guided by Handout 11: *Talk Frame Planning Template and Examples*, provide participants with an opportunity to consider what planning a task for implementing a Talk Frame might look like.

The handout provides two examples: the Flower Pattern Task and the prompt *What is 1 ÷ 2/3?* where students were expected to reason about the answer.

Additional examples for middle and high school prompts, including planning templates and student work for some prompts, are included in a supplemental handout *Mod3 Addtl Resources - ATOMIC 2014 Resources and Samples*. This set is from an ATOMIC presentation conducted by two project team members and targets middle and high school tasks primarily. (ATOMIC is the Associated Teachers of Mathematics in Connecticut.)

Several questions might prompt discussion at this point. For example:

- What makes for a "good" Talk Frame question or problem?
- What are some considerations for thinking about which student responses to have shared?
- What might be a good "we understand" for a prompt like comparing 4/6 and 6/10?
- How is the Number Talk routine like the Talk Frame routine?

Activity 3.4: Bridging to Practice Activity

As stated previously, the Bridging to Practice activities are a staple of this professional development that support participants to link the concepts of the PD with their work in classrooms and schools.

For the Bridging To Practice work, we encourage you to provide your participants with sets of Talk Frame icons, which can be found in the Bridges Task & Tool Repository on our website: http://bridges.education.uconn.edu/2015/06/19/allgrades_talkframe_iconsboard/
To make these icons more durable and versatile, we print and laminate them, and then affix square magnets to the back. Please note that a fourth "idea" icon has been included, labeled *Zani's Idea.* The purpose of this icon is to allow the teacher to introduce an idea at any point, labeled Zani's idea, for discussion. This is important when a teacher needs to ensure a particular response or idea is present for the discussion. (Note that another name could be substituted for Zani.)

Monthly PLC Format

For the PLC format, we encourage you to design activities that support participants to: (a) continue to think about the ideas already presented, (b) try out some ideas in a classroom setting with students, and/or (c) seed ideas for discussion in subsequent sessions.

One option for a Bridging to Practice activity between Module 3 and Module 4 is for participants to implement an argumentation task with students, and specifically one using the Talk Frame or another similar routine.

You could ask participants to do the following:

- Select or create a prompt for a Talk Frame. Plan, implement, and then reflect on how the implementation went. Record any questions they'd like to bring back to the group.

These reflections and questions can be part of an online conversation, emailed in advance to the facilitator(s), or shared at the beginning of Module 4.

As a second option, you might ask participants to revisit a task they implemented previously.

- Think about the argumentation task you did with your students after Module 2 (or equivalent). Is there anything you'd like to change or modify? Re-do the task with modifications or choose to implement a new argumentation task with a focus on 1-2 teacher and/or student practices discussed during Module 3 (e.g., norms, behaviors, Talk Frame, other routines).
- Note any changes you observed in student responses, peer-to- peer conversations, and/or student work as a result of modifications you made to your teaching practices or the task. If possible, videotape a class or a small group discussion.

Workshop Format – Talk Frame Mini-Lessons

For the intensive five-day workshop format, we outline here an activity (~ 70 minutes) for participants to do in teams. During this time, participants work in small groups to practice implementing a mini-lesson using the Talk Frame Routine.

To ease the transition into this activity, we encourage you to assign participants to teams ahead of time. Ideally, create an even number of groups so partnering groups can each have a chance to implement the Talk Frame with the other group as their students. Once participants are in their teams you provide them with the handout *3Bridging_Mini Talk Frame Lesson* which explains the activity, and then their page (only) of potential tasks to select from. We have included a (combined) set of tasks *3Bridging_Mini Talk Frame Tasks* – enough for 6 teams. This handout includes tasks for elementary, middle and high school. You will need to adapt this and adjust the questions depending on how many teams you have, and the grade levels taught by each team.

As a team, participants will select one of the tasks from a set of tasks provided. Then, participants will each work through that selected task individually. Then, again as a group, participants will discuss the problem focusing on:

- What's worth discussing related to this problem?
- What would be an important goal or *We Understand* for a Talk Frame discussion of this problem?
- Are there any modifications we want to make to the task to encourage argumentation as part of a Talk Frame Routine?

After this discussion, participants work with their team, using the Talk Frame Template to prepare a mini-lesson (15 minutes). When planning, participants should:

- Consider student friendly language for the focusing question/problem
- Anticipate ideas that may come up
- Record a potential goal (or goals) of discussion, which will be capture during the We Understand phase
- Decide how to teach this mini-lesson (who will do what, etc.)

Finally, each team will teach its mini-lesson to one (or more) of the other team and vice versa.

Similar to how we debriefed Activity 3.3 with the Talk Frame Routine, after both teams in the pair have had the opportunity to teach their mini-lesson, participants should discuss the

pedagogical aspects of this activity. Our debriefs were wide ranging, discussing the mathematics, choices made during the implementation, and questions about practice. This can also be followed by a whole group debrief.

Activity 3.5: Closure

Before the close of the session we found it was important to wrap up the ideas we discussed and provide some summary of the big ideas teachers should take away with them. Handout 12: Reflection on Norms has been included as a potential component of closure. (This handout is optional, or you might offer to participants as "thought questions." Some of the closure time can also be used to get feedback from participants both in order to (a) see what they are understanding from the material and (b) get information about how the facilitation, session organization, etc., are working for participants.

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Additional Resources: Module 3

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Wagganer, E. L. (2015). Creating math talk communities. *Teaching Children Mathematics*, *22*(4), 248–254. doi:10.1007/BF02985835 NCTM has also made this article freely available. You can find it at http://www.nctm.org/Publications/Teaching-Children-Mathematics/2015/Vol22/Issue4/Creating-Math-Talk-Communities/

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