Module 3:
Prompting and Supporting Argumentation – Focus on Implementation: Norms and Routines

Module Goals:

- Develop a deeper understanding of argumentation and its potential in the math classroom.
- Examine norms and routines that can support mathematical argumentation in the classroom.
- Develop an understanding of a pedagogy of inquiry to support mathematical argumentation in the classroom.

The formulation of a problem is often more important than its solution, which may be merely a matter of mathematical or experimental skill.

- Albert Einstein

BRAINSTORM

What classroom norms support a culture of thinking and mathematical argumentation?

Class Background for Video Clip: Establishing Norms for Argumentation

The students in the clip are six 6th-grade students to whom Ms. McKnight provided support and intervention several days a week in their math class. On Fridays, Ms. McKnight pulled the students altogether to another room to focus on Common Core Mathematical Practice 3. During these sessions students also worked with Sharon Heyman, a UConn graduate student working on the Bridges Project. These students were struggling in math at the time. Ms. McKnight's focused her instruction around argumentation and critiquing the reasoning of others beginning in late January. The clip is from a lesson near the end of March.

Ms. McKnight wrote the following to give a sense of her class and instruction around the video clip:

My initial goal was to engage students in forming a mathematical argument through discourse and writing. As with many students who struggle with math, these students hesitated to engage in mathematical discourse. So, I decided we needed to create a community agreement to set some guidelines. Because the students had very little experience in mathematical discourse my focus became more about strengthening mathematical discussion by modeling what this process should look like in a typical 6th-grade class. Sharon and I felt that getting them to engage in discourse and critiquing the reasoning of the others was going to be crucial to have them start the process of writing an argument.

After having the students create their community agreement, we showed them a video of other 6th grade students and asked them to notice what the students were doing and what the teacher was doing. They noted that the students did most of the talking and the teacher mainly asked questions. The second step was to model how discourse between two people who may have solved a problem with different strategies might look. I had the students use a discourse checklist to "score" the math discussion Sharon and I had after we solved a mathematical argument problem. We continued to use this checklist during our Friday sessions for student-to-student discourse. The third step was for me to be mindful about being explicit with my mathematical vocabulary and asking students many clarifying questions while they worked together in pairs or shared their thinking to the entire class. I also reminded the students that I was not looking for them to explain step-by-step how they solved a problem, but why they solved it the way they did. At this point, my focus was providing an arena for them to feel safe enough to discuss their math thinking. Over time we looked at ways they could help each other arrive at a correct answer.

It is also important to note that each session began with a type of "energizer" task that encouraged students to problem solve, reason and engage in mathematical discourse. This might have been a number talk, "can this be true?" problem, or critiquing an incorrect answer from previous sessions.

When we first started these Friday sessions, several things were apparent. The students had misconceptions about how to solve some of the problems in multiple ways, limited ability to justify their answers, didn't know how to explain their answer (the work) verbally. If they got a problem wrong, often they didn't look for how their answer was incorrect and then make adjustments. They hardly ever critiqued the reasoning of others. By the end of the school year, however, I saw lots of growth in almost all six students. One student, Chamari, highlighted in the video clip, showed the most growth in so many ways. Both Sharon and I saw his confidence level grow - he was able to clearly state how he solved a problem, he frequently shared his thoughts regarding the others students' solutions, and he was starting to show progression in his math class.

Video Viewing:
Establishing Norms for Argumentation
Guiding Questions

Please feel free to jot notes here and/or mark on the transcript.

1. What moves do you see the teacher making to help promote a culture of thinking?

2. What do you see students saying or doing that relates to a culture of thinking?

3. What norms may have been previously established in this class to support this interaction?

Ms. McKnight's Classroom Video Clip
Exploring Classroom Norms

1 Ms. M: So do you want to go first Chamari?

2 Chamari: I can't draw well.

3 Ms. M: You did a great job. So, Marion and Ari, now it's time to listen. You can draw on the board, I'll give you time to do that before you leave. Ari?

4 Ari: Yeah?

5 Ms. M: Be respectful, please. I said I'll give you time. So, can you guys find a place to sit? And Chamari is gonna talk about his idea. Okay. So, Chamari can you make sure all of us can hear. So, just tweak your body a little, and look at us.

6 Chamari: [turns to face whiteboard]

7 Ms. M: Look at *us*.

8 Chamari: [Turns around to the other side and looks out towards the class]
Okay. First, I drew the rect– I drew the boxes. Then, I…

9 Ms. M: So – I'm gonna – I'm only stopping you… And this is why I'm stopping him, because remember, we're going to try to stay away from telling step-by-step what we did. We want to talk about *why* we did that. So, why did you decided to do what you did? Instead of telling us step-by-step, because we didn't see what you did. How does it compare six-tenths – what you did – compare six-tenths and four-sixths?

10 Chamari: I chose this way because I thought it was easier. Because if I use the one bar then it would be easier to draw the denominators.

11 Ms. M: Okay. Can you hear him? Please stop drawing. Go ahead.

12 Chamari: Because… Like, if I drew – if I drew – use the one bar, then it would b easier to um… draw the denominators.

13 Ms. M: What about those denominators? What can you tell me about those denominators?

14 Chamari: Ah… They're both different, but they're both the same um… size.

15 Ms. M: So they're both different. Everyone agree with the denominators are different?

16 Student 1: I can't see.

17 Ms. M: Well, the denominators are the same as what you have. Six-tenths and four-sixths. Ten and six. Right?

18 Student 1: I said, but yeah.

19 Ms. M: Okay. Are they the same size? The denominator tenths and the denominator sixths? Are those the same size?

20 Students: No

21	Ms. M:	[Walks over to pick up fraction bar manipulatives] So if I look at some fraction bars – a sixth and a tenth – are those the same size?
22	Students:	No.
23	Ms. M:	They're not. So would you still stick with that, Chamari? That they're same size? So, they're different, right? We know that, for sure. […]
24	Ms. M:	Alright. So, what was your conclusion? I see that you wrote down six-tenths. What is your conclusion? Which one's greater? Which one's less? Or are they equal?
23	Chamari:	Ah, four-sixths is greater because four is closer – closer to the denominator… And six is further.
25	Ms. M:	Okay. So let me just make sure I understand. You're saying that four-sixths is the bigger fraction – it's greater. Does anyone know how I would write four-sixths is greater?
26	Chamari:	Yeah.
27	Ms. M:	So. Does anyone know how I would write four-sixths is greater?
28	Ari:	Yes.
29	Ms, M:	Can you come up and show me? And so, Trinity and Michael and Jason, can you look over her?
30	Ari:	[Walks to the whiteboard and fills in the symbol so the comparison looks like: $\frac{6}{10} < \frac{4}{6}$]
31	Students:	No. Yes.
32	Ms. M:	So, who can tell me which one says that four-sixths is greater? The top one or the bottom one?
33	Student 2:	Bottom.
34	Ms. M:	Why?
35	Student 2:	Because, the – where the…
36	Ms. M:	So, this is a symbol.
37	Students:	[overlapping explanations]
38	Ms. M:	Yeah. And we can make it as simple as that, right? We can say… And that's just how I remember it. Like, oh yeah, this is like the bigger size … And like kind of chomping down on the bigger. I think you're thinking – I know what you're thinking here. You just have know how to write the symbol. Okay? So, who else wants to explain or maybe agree or disagree? Chamari is saying that six-tenths is less than 4/6. [Class discussion continues]

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This document describes general norms and expectations about mathematics that can help to promote a culture of thinking in math class. The last page offers a progression of expectations and taught behaviors for supporting argumentation in an elementary classroom.

Positive Norms to Encourage in Math Class

By Jo Boaler

http://www.youcubed.org/wp-content/uploads/Positive-Classroom-Norms2.pdf

1. Everyone Can Learn Math to the Highest Levels.
Encourage students to believe in themselves. There is no such thing as a "math" person. Everyone can reach the highest levels they want to, with hard work.

2. Mistakes are valuable
Mistakes grow your brain! It is good to struggle and make mistakes.

3. Questions are Really Important
Always ask questions, always answer questions. Ask yourself: why does that make sense?

4. Math is about Creativity and Making Sense.
Math is a very creative subject that is, at its core, about visualizing patterns and creating solution paths that others can see, discuss and critique.

5. Math is about Connections and Communicating
Math is a connected subject, and a form of communication. Represent math in different forms eg words, a picture, a graph, an equation, and link them. Color code!

6. Depth is much more important than speed.
Top mathematicians, such as Laurent Schwartz, think slowly and deeply.

7. Math Class is about Learning not Performing
Math is a growth subject, it takes time to learn and it is all about effort.

Some Additional Resources for Norms and Expectations in the Mathematics Classroom

This handout contains excerpts from three resources focused on supporting discussion and reasoning in mathematics classrooms.

1) Excerpt from *Classroom Norms for Productive Discourse & Discussion*
(O'Connor & Ruegg, 2012)

Classroom discourse and discussion are crucial sites for student development: they can help students develop the ability to be critical thinkers, and to feel more confidence in their own abilities to construct arguments and to understand the arguments of others.

Nevertheless, classroom talk and discussion can be challenging. Teachers who tend to have success with classroom discourse and discussion usually have worked out dependable ways to make sure that the discourse is *respectful, equitable*, and *focused on reasoning*. They make sure that they set up norms of productive discussion within the first week or so of school. This usually includes more than posting norms on the wall. It usually includes a discussion about what is involved, to get student understanding and buy-in.

What is included in setting up norms for discussion?

In setting up norms, you will be letting your students know about how talk and discussion will play a role in their learning during the coming year. You are setting up expectations, and so your students need to understand those expectations. Depending on the ages and stages of your students, it is most helpful if you can enlist their participation in co-constructing those expectations. Consider having a discussion about these three different kinds of discussion norms and goals:

1) Talk that is focused on *reasoning*
2) Talk that is *respectful*
3) Talk that is *equitable*

Here is an example of norms for discussion that an eighth grade teacher co-created with her students:

- We come prepared for discussions with notes, examples, stories, texts.
- We are active participants, responsible for our own learning. This means we speak, request clarification, show agreement or confusion, verify, ask others to repeat.
- We strive to have authentic discussions that are academically rich. This means we stay on topic and ask what we really want to know.
- We push ourselves and each other to think beyond the obvious, popular, or easy answers. This means we request proof or reasoning, point out misinformation, disagree with parts, draw others out, and are open to changing our minds.

2) Excerpt from "Revoicing: The good, the bad, the questions" by Jean Krusi (High School Algebra Teacher) in the collection *Promoting Purposeful Discourse* (Herbel-Eisenmann & Cirillo, 2009, p. 121)

As I thought about how I wanted to change the discourse patterns in my classroom, I decided I wanted to involve my students in the process. I started by asking them what made a good classroom discussion. The students seemed to be importing ideas about class discussion from their other classes, possibly indicating that they did not think of mathematics classes as places for rich discussion. We used the ideas of what makes a good discussion, more generally, to produce the following set of discussion norms for our classroom:

- Everyone is listening. Everyone is involved. Everyone puts out ideas. No one is left out.
- No one is talking while someone else is. Take turns.
- Questions are asked. Make your point clearly and quickly. Have facts to back up your point. It is safe to be wrong.
- No rude comments or put-downs. All ideas and opinions are respected. Different points of view are valued. "Out of the box" thinking can be helpful.
- Everyone is understanding-if not at the beginning, then by the end.

Most of these norms are social norms. Some, such as "have facts to back up your point" and "everyone is understanding," do have the potential to suggest mathematical norms. During our discussions, I found that I needed to bring up mathematical norms, such as what counts as evidence in mathematics or the idea of generalizing statements. In the future, I would like to try having a discussion about elements particular to mathematical discourse as a class, with the idea of generating mathematical norms as well as social norms.

3) Excerpt from the book *Making Number Talks Matter* about how to make a safe classroom environment (Humphreys & Parker, 2015, p. 169)

Our bottom line is that we want the learning environment to be safe for all students. Ruth shares, "I tell my students on the first day of class that I won't put them on the spot but that I will give them lots of opportunities to share their thinking when they choose to. I try hard not to violate this trust. I want the learning environment to be safe for all students. I do talk with kids about how important it is for them to talk about and explain their thinking. With quieter students, I sometimes ask them, one-on-one, to share with me how they thought about a problem. Once they have had a chance to rehearse their thinking with me, I ask them to think about whether they might be willing to share their ideas. Once students have had their own way of thinking recognized and valued, they may become more confident in sharing their ideas."

Norms & Expectations for Argumentation in Partners
Progress Across the Year in a 3rd Grade Class

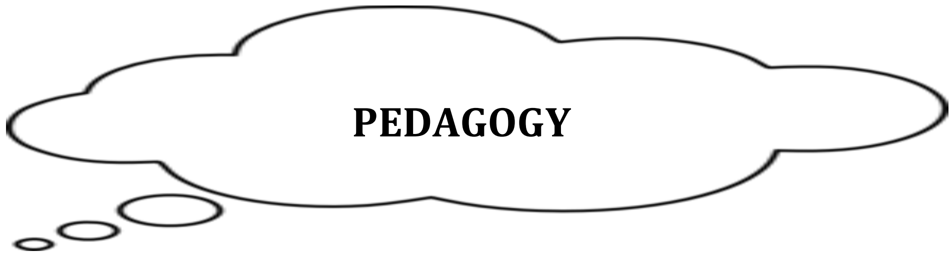
Early in the School Year	• Facing each other, hands and legs quiet. • Looking at each other. • One person shares their claim, evidence and warrant and the other person listens. • The other person can say or write exactly what the other person has just told them. • Repeat with the other person
	• One person shares their claim, evidence and warrant and the other person listens. • The other person can say or write in their own words and ask questions what the other person has just told them. • Repeat with the other person.
	• One person shares their idea and the other person listens. • The other person agrees or disagrees with the other person's claim, evidence and/or warrant and explains why. (Concentrate on agreeing or disagreeing with the argument, NOT the person.) • Repeat with the other person.
Later in the School Year	• Each person shares their claim, evidence, and warrant. • They ask each other clarifying questions. • They ask each other to explain their evidence in a different way. • They agree or disagree with each other and state why using their evidence. • They build upon what they are learning from their partner's argument and improve or change their own argument.

Questions a teacher can ask to help promote argumentation	
Can you explain?	What does that look like?
What happened before?	What happened after?
What would happen if you used this number?	Can you tell me why…?
What would change if...	Tell me more.
Show me where...	How does this fit?
What could you add to strengthen this part?	Why did you use that equation?
How would that work?	So you are saying…?

Modified based on resources created by Sarah Brown

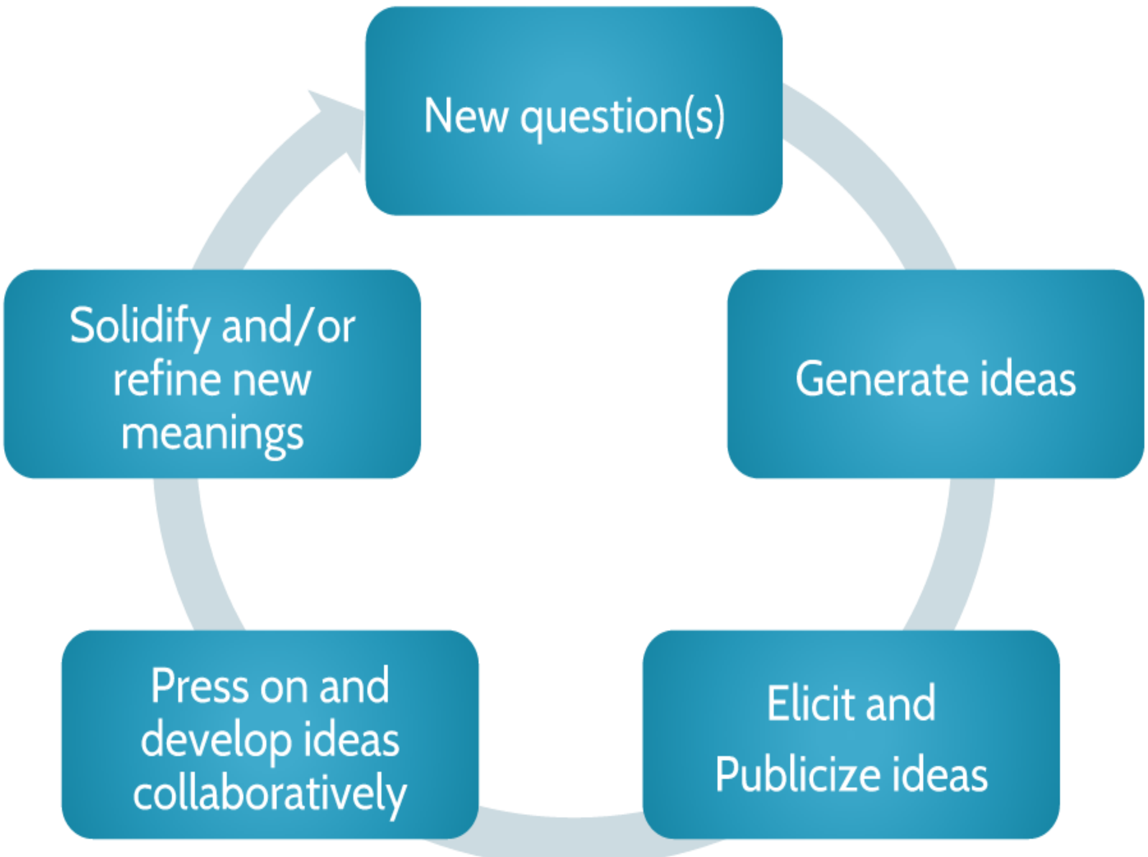
ATOMIC 2015 Presentation: *What is Mathematical Argumentation?*

Brainstorming About Routines



What routines or instructional strategies can be used in the classroom to support a culture of thinking and mathematical argumentation?

A Pedagogical Model to Support a Culture of Thinking



Math Task:
Chain of Flowers Pattern

Consider the following pattern:

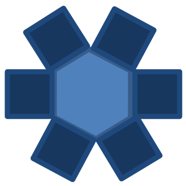


Figure 1

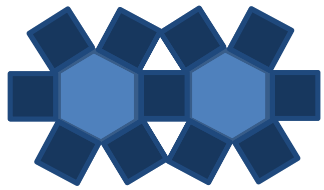


Figure 2

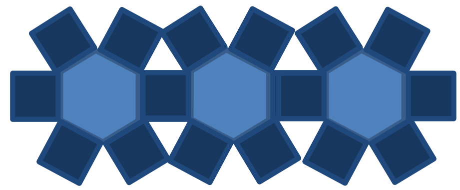


Figure 3

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a. Draw Figure 5. How many tiles does it have?

b. How many tiles will the 25th figure have? How do you know?

c. How many tiles are in the *n*th figure? How do you know?

Debriefing the Talk Frame Routine
Flower Pattern Task

These questions are meant to help prompt discussion and reflection. You do not need to discuss each, or each at equal depth.

1. How does the Talk Fame allow students to generate ideas?

2. How can the teacher facilitate this process?

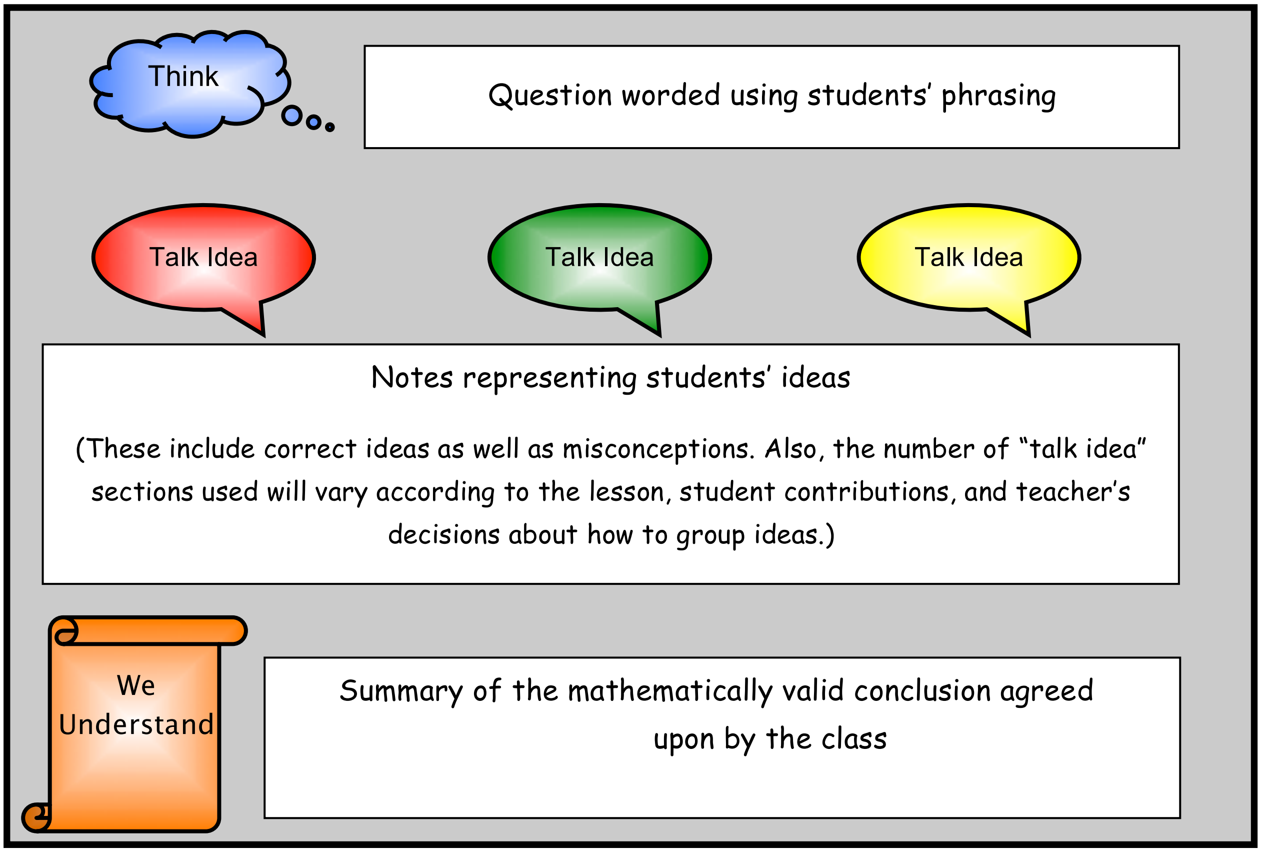
3. How does the Talk Frame help support a student-centered classroom? What role do student responses play?

4. What can the teacher do to help students make connections between ideas and solidify their learning?

5. How does that Talk Frame Routine support students' participation in mathematical argumentation?

Overview of the *Talk Frame* Pedagogical Routine

This *Talk Frame,* and other similar pedagogical "tools," is a routine that can be used to help organize mathematical discussions, keep track of student contributions, and synthesize the mathematical ideas. Note that the teacher can also introduce an idea using another idea card, not shown here.



From *Exploring Shape Games: Geometry with Imi and Zani*, by M. K. Gavin, T. M. Casa, S. H. Chapin, and L. J. Sheffield.

For teachers, the Talk Frame can:

- Help focus the discussion on a significant mathematical topic;
- Encourage you to concentrate on and make sense of students' ideas;
- Allow you to formatively assess the class's and individual student's depth of understanding; and
- Assist in facilitating the process for the class to come to valid mathematical conclusions.

For students, the talk frame can:

- Allow them to see their thinking develop over time;
- Encourage them to rely on their own reasoning;
- Convey that their ideas are important, yet they can change;
- Let them realize that there can be multiple perspectives when solving problems;
- Have them see how different representations, such as drawings, can symbolize ideas;
- Showcase the appropriate use of math vocabulary; and
- Reinforce the eventual need for mathematical agreement.

Talk frame. It can be challenging for both teachers and students alike to keep track of all that has been said during a discussion. Developed as part of the *Project M^2* units, a *talk frame* helps to keep track of student contributions and revisions to their ideas. The talk frame is a system that helps teachers organize and maintain a more permanent record of the conversation on the board. There are three sections to the talk frame using distinct icons:

1. Establish the topic: Teachers first need to identify the topic of discussion. It should be based on an important mathematical idea of the lesson and one for which students probably will be able to offer different ideas (either because there are multiple solution paths, various correct answers, or current misconceptions and misunderstandings among students).

2. Have students offer, consider and clarify ideas:
Teachers then need to gather different student perspectives. It is important for teachers not to judge their validity at this point, either verbally or through gestures, since students should be encouraged to make sense of the mathematical concepts themselves. Do not erase student ideas as they revise them. A record of the entire discussion should be produced. If necessary, teachers can introduce different perspectives they want students to contemplate through Zani (a character used in the *Project M^2* units). It is important that Zani introduces *both* correct and incorrect ideas; otherwise students may assume that anything presented by them is incorrect. Through this entire process, students will clarify their ideas both for themselves and others.

3. Settle on a mathematical understanding: Teachers eventually will guide students to reach a mathematical understanding. Often, students will notice the errors in their thinking given time to debate and justify their ideas. In this section, the new understandings are recorded. Teachers may elicit multiple "we understands" from students, or prompt the class to articulate one key understanding based on the discussion.

Talk Frame Planning Template

Think

Talk Idea

Talk Idea

Talk Idea

We Understand

____

____

____

____

____

Talk Frame Planning Template

Flower Pattern Task

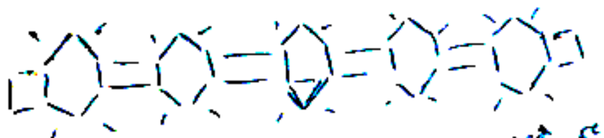
Think

- How many tiles in the 5^{th} figure?
- How many tiles in the 25^{th} figure?
- How many tiles in the n^{th} figure?

Talk Idea

Pictorial Representation:

a. 5^{th} figure



How many tiles?

5 hex + 6 horizontal squares + 4 · 5 slanted squares per hexagon = 31

Talk Idea

Numerical/Tabular Representation:

Figure	tiles
1	7
2	13
3	19
4	25
5	31

I got this by following the pattern. For figure 1, 2, 3, you see that it adds 6 each time. Following that pattern, 4^{th} is 25 and 5^{th} is 31.

Talk Idea

Algebraic Representation:

Expression $\Rightarrow n + 2n + 2n + (n-1) + 2$

(# of hex.) + (# of top □'s) + (# of bottom □'s) + (# of connecting □'s) + (beginning & ending □)

Equivalent Expression $5n + n + 1$

(# of □'s for n ⬡) + (# of ⬡'s) + (Last □ in set)

Talk Frame Planning Template

We Understand

- ***h*** represents the number of hexagons in the figure, and the number of squares can be written in terms of the number of hexagons.
- There are equivalent expressions to represent the solution
- The non-algebraic representations can be translated in to the algebraic.

Talk Frame Planning Template

Think

Question worded using students' phrasing:

Without using the traditional algorithm, can you make sense of $1 \div \frac{2}{3}$?

Talk Idea | Talk Idea | Talk Idea

Anticipated ideas

(Diagrams & pictures are likely also)

1/3 goes into 1 three times & 2 times 3 equals 6. So the answer is 6

Dividing by 2/3 is the same as multiplying by its reciprocal (3/2) That's the same as multiplying by 3 and then dividing by 2. 1 x 3=3 and 3 divided by 2= 1 ½

How many 2/3s fit into 1? On a number line, one 2/3 fits & then another 1/3 fits. 1/3 is half of 2/3, so one & a half 2/3s fit into 1

What's a potential goal(s) for the discussion? What should students come to better "understand"?

We Understand

- We can explain fractions division problems in ways that make sense.
- We can figure out if an answer (or method) to a fractions division problem is reasonable.
- We agree that 2/3 can fit into 1 one time with 1/3 left over and 1/3 is ½ of 2/3 so $1 \div \frac{2}{3} = 1\frac{1}{2}$

Question from Boaler & Humphreys (2005). *Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning.* Portsmouth NH: Heinneman.

Reflecting on Norms in Your Own Practice

1. Which norms/routines do you see yourself using at the beginning of the school year? Why?

2. All students can and should have opportunities to participate in argumentation. How might that work differ from class to class, or across groups of students with different needs?

3. What have you tried already that has worked well in your classroom?