Mazzotta & Staples

PACKET OF RESOURCES
Let's Talk About It
ATOMIC 2014

These materials were shared at the ATOMIC 2014 conference. The packet contains several agumentation tasks (secondary focus), with a planning template and/or student work samples.

List of Talk Frame Prompts included

Proportional Reasoning:
- Trees: Which Grew More (Lamon 2006)
- Which is More Square (Lamon 2006)
- Traffic Jam

Algebra I:
- Dog Kennels*
- Animal Populations*
- Hexagon Task*

Geometry:
- Midpoint*

Algebra II:
- Extraneous Solutions

Precalculus/Calculus
- Limits*

Those with an asterisk (*) include sample Talk Frame planning tool or student work samples

Back side of this sheet is a set of curricular and pedagogical resources.

Some resources - *Let's Talk About It*

Curricular materials to support CCSS, including argumentation

- www.map.mathshell.org - very useful resource for both tasks and fully developed lessons. The lessons are formative assessment lessons, with a pre-assessment activity, main learning activity, and post assessment. All are aligned to CCSS content and standards.
- Illustrativemathematics.org – site being developed to help make the CCSS document come to life through tasks and examples. Tasks often have "teacher notes."
- http://illuminations.nctm.org - very extensive set of well developed lessons. Searchable by grade and topic. There are some itunes aps for some lessons.
- www.nrich.maths.org - site out of England with really nice problem solving activities, challenge activities; organized by grade levels and "stages" to indicate difficulty. Some nice support materials like slides and short videos.
- http://www.mathshell.com/publications/tss/ppn/ppn_masters.pdf - a really nice set of materials from the Shell Centre in England. This PDF has a lot of problems focused on Number and Patterns.

Books, articles, etc., with a focus on supporting pedagogy of argumentation

Barlow, A. T., and McCrory, M. R. (2011). 3 strategies for promoting math disagreements. *Teaching Children Mathematics, 17*(9), 530-539. Summary of article at: *http://bestpracticesweekly.com/wp-content/uploads/2011/05/Math-Debate-Article.pdf*

*Casa, T. (2013). Capturing Thinking on the Talk Frame. *Teaching Children Mathematics. 19*(8), 516-523.

Chapin, O'Connor, & Anderson (2003). *Classroom Discussions: Using Math talk to Help Students Learning: Grade 1 – 6.* Sausalito, CA: Math Solutions Publications.

Ellis, A., Bieda, K. & Knuth, E. (2012). *Developing Essential Understanding of Proof and Proving: Grades 9 – 12,* Reston, VA: National Council of Teachers of Mathematics.

Hoffman, B., L. Breyfogle, & J. A. Dressler (2009). The power of incorrect answers. *Mathematics teaching in the middle school, 15*(4), 232-238.

Knudsen, J. & Lara-Meloy, T. (2014). Advice for mathematical argumentation. *Mathematics teaching in the middle school 19*(8), 494-500.

Three middle school teachers engage students in argumentation by *telling* and *showing* them how to argue. A mathematical graphic organizer with three sections (Conjecture, Justification, and Conclusion) was used to help students organize their mathematical arguments.

Lannin, J., D Barker, & B. Townsend (2004). "Why, why should I justify?" *Mathematics teaching in the middle school, 11(*9), 438-443.

Smith, Margaret S. and Mary Kay Stein. *5 Practices for Orchestrating Productive Mathematics Discussions.* Reston, VA: National Council of Teachers of Mathematics, 2011.

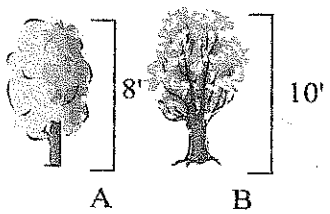
Stockero, S. L., Zoest, L. R. V., Kinzel, M., & Cavey, L. (2011). Making student thinking public. *The Mathematics Teacher, 104*(9), 704–709.

Bridging Math Practices website

http://bridges.education.uconn.edu Website with materials from PD grant focused on argumentation, proportional reasoning, and pedagogy. Materials and resources under Summer Workshop tab and Academic Year Workshop tab

The Trees Problem
from Lamon (2006)

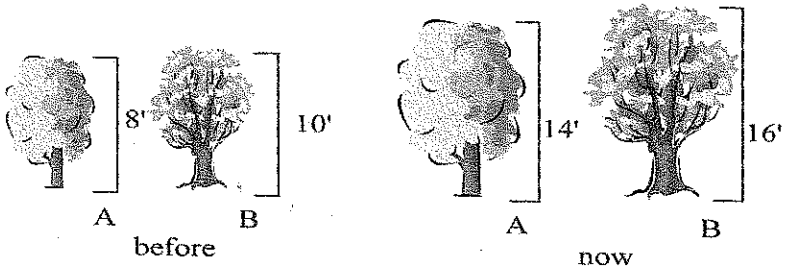
1) (a) Which tree is taller?



(b) How would you describe **how much taller**?

Think

2. Before, tree A was 8' tall and tree B was 10' tall. Now, tree A is 14' tall and tree B is 16' tall. Which tree grew more? Explain your reasoning.



3. (a) Write down new ideas that were uncovered from talking with others.

We Understand

(b) *We understand...*

Think

Which tree grew more? Explain your reasoning.

We Understand

Talk Idea

16 - 10 = 6

14 - 8 = 6

They grew the same amount of feet

The same

Zani's Idea

Talk Idea

Ditto

A: $\frac{6}{8} = 75\%$

B: $\frac{6}{10} = 60\%$

Tree A grew prop. more

Talk Idea

Tree A

Almost double its size

B was "further away from dividing"

Tree A's orig height is 57% of its new height.

Tree A
8 orig
4 "new"
.57

Tree B
10
16
.62

Tree B's orig height is 62% of the new height

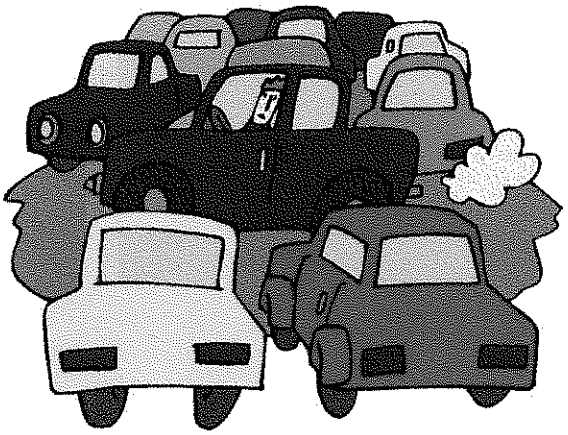
Square-ness Task

(from Lamon, 2006)

Which is *more square*, a rectangle that measures 35' x 39" or a rectangle that measures 22" x 25"?

Create a mathematical argument to explain your answer.

Traffic Jam Problem:



Last Sunday two news stations reported an accident that caused a 12 mile traffic jam on a straight stretch of a two-lane freeway.

How many vehicles were in the traffic jam?

Defend your answer with a mathematical argument.

TALK FRAME Template Algebra 1 Equation intro

Think

Doggie Land Kennel charges a fee of $25 and then $40 for each night that you board your dog.
Bark and Wag Kennel charges a fee of $40 and then $35 for each night that you board your dog.
For how many nights will the costs of the kennels to be equal?

Give a mathematical argument to support your answer.

Talk

Algebraic Equations

1. C=25+40x
2. C=40+35x
3. 25+40x=40+35x

Talk

Make a table for each Kennel and find out how much each day costs at each kennel until you find a day where they cost the same

Talk

Guess and check. Substitute numbers in until you find an answer.

(maybe also a graphing response-but I don't foresee that with the numbers given)

We Understand

- We understand that there are different ways to solve this problem.
- We understand that quantities may be compared numerically, algebraically, or in an application.
- We understand that we can use the information given to set up mathematical equations.

Name_____ Date_____

Algebra 1
Equation Intro

Doggie Land Kennel charges a fee of $25.00 and an extra $40.00 for each night that you board your dog.

Bark and Wag Kennel charges a fee of $40.00 and an extra $35.00 for each night that you board your dog.

How many nights would you have to board your dog for the cost of each kennel to be equal?

Give a mathematical argument to support your answer.

Name Student A Date

Algebra 1
Equation Intro

Doggie Land Kennel charges a fee of $25 and then $40 for each night that you board your dog.
Bark and Wag Kennel charges a fee of $40 and then $35 for each night that you board your dog.
For how many nights will the costs of the kennels to be equal?

Give a mathematical argument to support your answer.

DL: 25 + 40 per night BW: 40 + 35 per night

Nights	DL	BW
1	25+40=65	40+35=75
2	+40/ 105	+35/ 110
3	+40/ 145	+35/ 145 ← They will be the same cost for 3 nights.
4	+40/ 185	+35/ 180
5	+40/ 225	+35/ 215

The table shows the values for each company. They are the same cost for 3 nights.

Name Student B Date

Algebra 1
Equation Intro

Doggie Land Kennel charges a fee of $25 and then $40 for each night that you board your dog.
Bark and Wag Kennel charges a fee of $40 and then $35 for each night that you board your dog.
For how many nights will the costs of the kennels to be equal?

Give a mathematical argument to support your answer.

1 night ⇒ Doggieland = 25 + 40 = $65
Bark and Wag = 40 + 35 = $75 ✓ costs more

4 nights ⇒ DL = 25 + 40 + 40 + 40 + 40 = $185 ✓ costs more
BW = 40 + 35 + 35 + 35 + 35 = $180

3 nights → DL = 25 + 40 + 40 + 40 = $145
BW = 40 + 35 + 35 + 35 = $145
] They are the same for 3 nights.

For one night Bark and Wag costs more and for four nights Doggieland costs more. So the I checked between them at three nights and they were equal.

Name Student C Date

Algebra 1
Equation Intro

Doggie Land Kennel charges a fee of $25 and then $40 for each night that you board your dog.
Bark and Wag Kennel charges a fee of $40 and then $35 for each night that you board your dog.
For how many nights will the costs of the kennels to be equal?

Give a mathematical argument to support your answer.

DL → Cost = 25 + 40n

BW → Cost = 40 + 35n

To find out where they are equal we solve them

$25+40n = 40+35n$
$-35n \quad -35n$

$25+5n = 40$
$-25 \quad -25$

$\frac{5n}{5} = \frac{15}{5}$

$n = 3$

My equations and work shows that they will be the same cost for 3 nights.

Talk Frame - Planning Tool

Grade level: Algebra | Topic: Animal Populations

Focus (content standards and/or SMPs): seeing structure in expressions

Think

Suppose P and Q give the sizes of two different animal populations, where Q > P. In (a) – (d), say which of the expressions is larger. Briefly explain your reasoning in terms of the two populations.

(c) $\frac{P}{P+Q}$ and $\frac{P+Q}{2}$

Talk Idea

[algorithmic approaches; symbolic manipulation to compare]

Student compares by finding a common denominator of 2(P + Q)

Student compares by cross multiplying

Talk Idea

[reasoning about quantities using numbers]

Student compares by picking valid value for P and Q.

Student compares by picking valid P and Q, and then arguing relationship of expressions always

Student compares by comparing each expression to an "anchor" (e.g., to 1)

Talk Idea

[reasoning about the quantities the expressions represent]

Student compares by interpreting each expression as a quantity in the context of the problem – e.g., P/(P+Q) is the **proportion** of the population of animal P to the total population (of P & Q combined) (so this is less than 1) and (P + Q)/2 is the **average** of the two populations (so this is more than 1)

We Understand

We understand there are multiple ways to compare quantities – numerically, symbolically, and by considering what the quantities represent in the context.

We understand that plugging in a valid P and Q helps, but that an argument for which quantity is bigger needs to account for all relevant P and Q values. We must make the argument general (for all Q > P)

ANIMAL POPULATIONS

(C) $\frac{P}{P+Q}$ and $\frac{P+Q}{2}$

6 potential student approaches/arguments

(1) Compare both expressions to 1

answer: $\frac{P+Q}{2}$

reasoning: $\frac{P}{P+Q}$ is <1 because the numerator (P) is less than the denominator ($P+Q$) so the fraction must be less than 1.

$\frac{P+Q}{2}$ is >1 because P & Q are positive integers, so $P+Q \geq 2+1$ and so $\frac{P+Q}{2} \geq \frac{3}{2} > 1$.

(2) Compare expressions using a common denominator

$\frac{P}{P+Q}$? $\frac{P+Q}{2}$

get a common denominator

$\frac{2P}{2(P+Q)}$? $\frac{(P+Q)^2}{2(P+Q)}$

→ Compare

– now need only consider numerator

$2P$? $(P+Q)^2$

$2P$? $P^2+2PQ+Q^2$

$2P$ < $P^2+2PQ+Q^2$

So $\frac{P+Q}{2} > \frac{P}{P+Q}$

OR

Cross multiply

(3) using a sample pair of numbers.

note: for this to be a valid approach, need to explain how this test case represents all relevant cases or the general case.

To show the relationship, test with specific values,

let $P = 5$

$Q = 5^+$ (some number larger than 5)

$\frac{5}{5+5^+} < \frac{5+5^+}{2}$.

$\frac{5}{5+5^+}$ ↑ less than $\frac{1}{2}$

$\frac{5+5^+}{2}$ ↑ more than 5

(4) Same as #3, but the symbolic version of it

let $P = n \in \mathbb{Z}$, let $Q = n+1$ (or $n+x$)

$\frac{n}{n+(n+1)}$? $\frac{n+(n+1)}{2}$

$\frac{n}{2n+1}$? $\frac{2n-1}{2}$

which is larger?

this is less than 1.

$n+1/2$, so this is $> n$ and $n > 1$ (or $> P$)

(5) use context

$\frac{P}{P+Q}$ is the proportion of the population (P) to the combined pop ($P+Q$), so this, when $\times 100$, is the percent of the population

$\frac{P+Q}{2}$ is the average population for the 2

a percent of the population is less than an average portion.

(6) MISCONCEPTION

$\frac{P}{P+Q}$? $\frac{P+Q}{2}$

$\frac{P}{P}+\frac{P}{Q}$? $\frac{P}{2}+\frac{Q}{2}$

$1+P/Q$? $\geq 1/2 \quad \geq 1$

since $P<Q$ $1+\frac{P}{Q}<2$

here, adding we have $> 3/2$ and it can grow

so $\frac{P+Q}{2}$ is larger

Hexagon Task

Each figure in the pattern below is made of hexagons that measure 1 centimeter on each side.

Figure 1
Perimeter = 6 cm

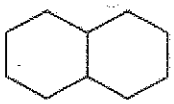


Figure 2
Perimeter = 10 cm

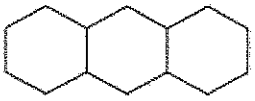


Figure 3
Perimeter = 14 cm

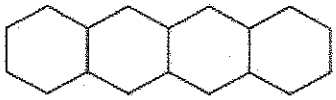


Figure 4
Perimeter = 18 cm

1. Draw and find the perimeter of Figure 5.
2. If the pattern of adding one hexagon to each figure is continued, what will be the perimeter of the 25th figure in the pattern? Justify your answer.
3. Extension: How can you find the perimeter of *any figure.* (A figure with *n* hexagons?)

Student Work Samples – Hexagon Task – Perimeter of the 25th figure

Student A

So we multiplyd 25x4 added two then we got 102.

Student B

102cm for 25 figures

I got 102cm because I used a table. I choose a table because you add 4 to the perimeter every time

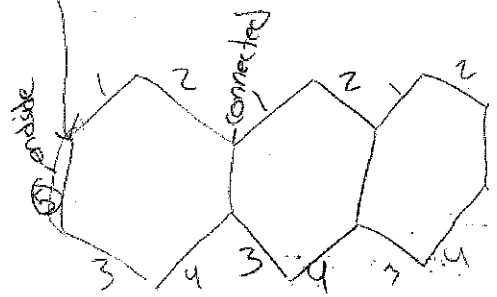
1	6	>+4
2	10	>+4.
3	14	
4	18	
5	22	
6	26	
7	30	
8	34	
9	38	
10	42	
11	46	
12	50	
13	54	
14	58	
15	62	
16	66	
17	70	
18	74	
19	78	
20	82	
21	86	
22	90	
23	94	
24	98	
25	102	

Student C

calculations or statements to the diagram. The perimeter of the 25th figure in the pattern is 102 because when you multiply 25x4 you get 100. I got the 25 because of the 25 hexagons and I get the 4 because that is how many each hexagon has not including the two end sides. Then I would add on 2 because of the two end sides. When I add it all up I get 102.

1x4+2=answer

25 hexagons
* 4 sides
100 sides
+ 2 end sides
102 sides



Student D

Student E

I noticed a pattern, the pattern was that every time you added on a hexagon, so I decided to add 4 to 18 and got 22 then I multiplied 22x5 because 5x5=25 and 22 is the perimeter of five hexagons together X 5 would equal the perimeter of 25 hexagons. So I got 110 cm as the perimeter for 25 hexagons.

Student F

Each shape has 6 sides, but all but one of the right sides are on the inside, and all but one of the left sides are on the inside

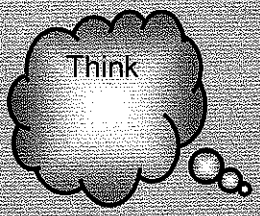
6·25=150
150-24=126
126-24=102
the answer is 102

Student G

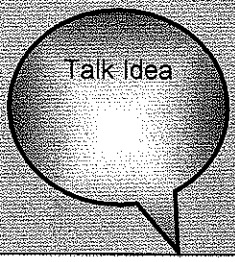
P=n·4+2
P=25(4)+2
P=100+2
P=102

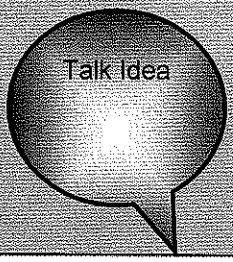
25th Figure
P = 102 cm

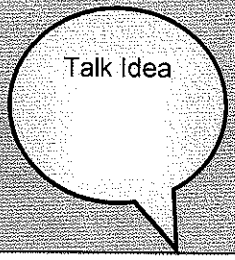
Talk Frame – Mid-Point



1. Find the point exactly in the middle of points A and B.
2. Construct an argument to convince me that your point is precisely in the middle.

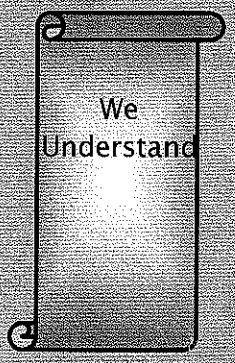






Anticipated ideas

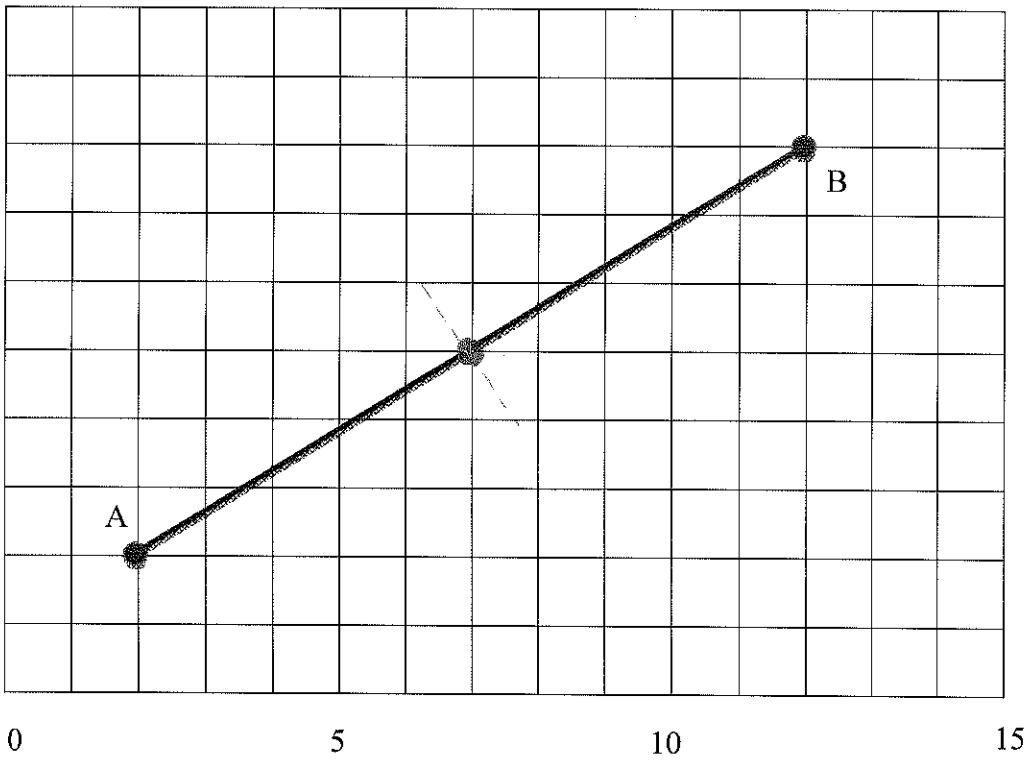
1. You can use a ruler and measure finding the point that cuts the line (segment) into two equal parts
2. You can fold the line (by folding the paper) to line up the points and mark the line on the crease with a point
3. You can solve it algebraically using the mid-point formula
4. You can put a point on the line that looks like its in the middle.



- It's a point called the mid-point
- It's equidistant from points A & B (it bisects the line segment)
- It can found using multiple methods
- The measures of the two halves are equal
- The two 'half' line segments are congruent

Talk Frame – Mid-Point

Name(s): Student A Date: _____

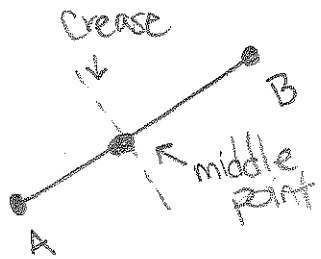


1. Find the point exactly in the middle of points A and B.

2. Construct an argument (words, drawings, graphs, etc) to convince me that your point is *precisely* in the middle.

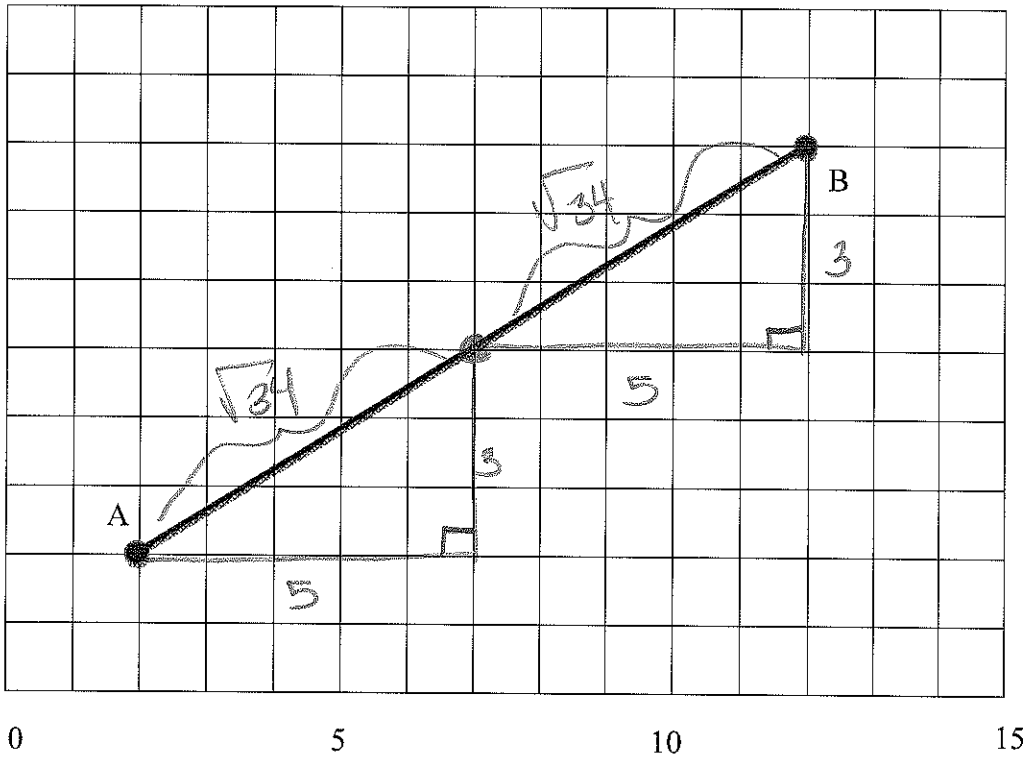
My point is precisely in the middle because it is half way between points A and B. I know this because I folded my paper so that points A and B lined up, and the point in the middle is on the crease. The middle is exactly the same distance from A as it is from B, so my point is precisely in the middle.

Room for sketches if necessary



Talk Frame – Mid-Point

Name(s): Student B Date: _____

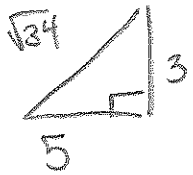


1. Find the point exactly in the middle of points A and B.

2. Construct an argument (words, drawings, graphs, etc) to convince me that your point is *precisely* in the middle.

My point is at the middle of the segment because it is the same distance from A as it is from B. I used the Pythagorean Theorem to find the distance between A and the middle, and the distance between B and the middle. I got the same answer for both: $\sqrt{34}$ units. Since both distances are the same, my point is precisely in the middle.

Room for sketches if necessary



$5^2 + 3^2 = c^2$

$25 + 9 = c^2$

$\sqrt{34} = \sqrt{c^2}$

$\sqrt{34} = c$

$5^2 + 3^2 = c^2$

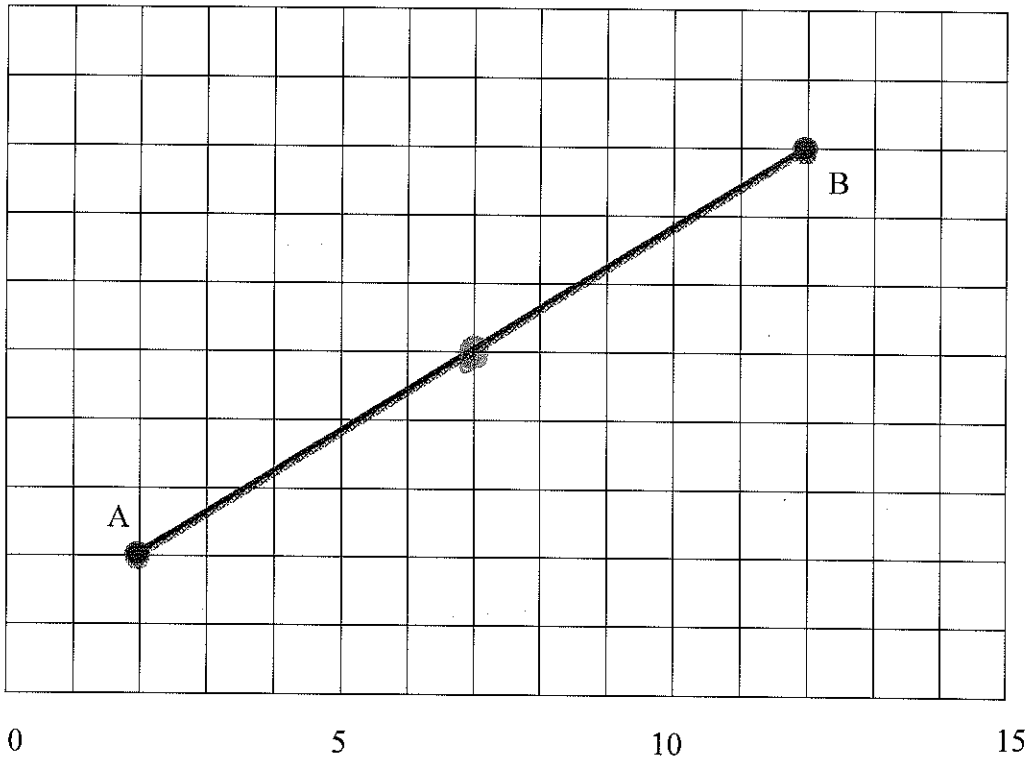
$25 + 9 = c^2$

$\sqrt{34} = \sqrt{c^2}$

$\sqrt{34} = c$

Talk Frame – Mid-Point

Name(s): Student C Date: _____



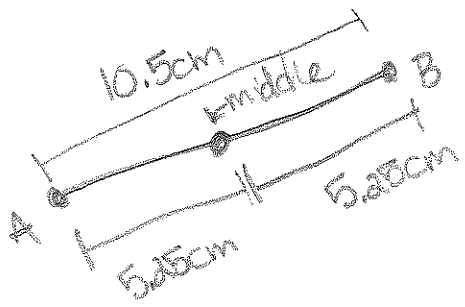
1. Find the point exactly in the middle of points A and B.

2. Construct an argument (words, drawings, graphs, etc) to convince me that your point is *precisely* in the middle.

My point is exactly in the middle. Using a ruler, I found that the segment AB is 10.5cm long. Since the middle is half way, I can divide the length in two to get 5.25cm. I then measured 5.25cm from A and 5.25cm from B along the segment and got the same point both times. My point is the middle of the segment.

Room for sketches if necessary

$\frac{10.5\text{cm}}{2} = 5.25\text{cm}$



Talk Frame - Planning Tool

Grade level: Algebra 2 Topic: _Extraneous Solutions_____

Focus (content standards and/or SMPs)

Think

Joe and Cindy are asked to do the following math problem.

Find the intersection(s) of $y_1 = \sqrt{x-1} - 3$ **and** $y_2 = x - 10$.

Joe says the answers are x=10 and x=5. **Cindy** says the answer is x=10. Who do you agree with and why?

Talk Idea

Student solves by graphing – gets only the solution of x = 10. Agrees with Cindy.

Talk Idea

Student solves by setting equations equal to one another. Finds 2 solutions (may or may not realize that one is extraneous). Agrees with Joe (if doesn't check) or Cindy (if checks)

Talk Idea

Student solves by plugging in values of x = 10 and x = 5 and checks to see if they work. Agrees with Cindy.

We Understand

Focus on what it means to solve an equation

Follow up question: Why does the algebraic approach produce an additional solution (that's not a solution at all)? (Discuss step where square both sides.)

_____

_____

Name: _____ Date: _____ Per: _____

Critiquing the Arguments of Others

Joe and Cindy are asked to do the following math problem.

Find the intersection(s) of $y_1 = \sqrt{x-1} - 3$ **and** $y_2 = x - 10$

Joe says the answers are x=10 and x=5.

Cindy says the answer is x=10.

Who do you agree with and why?

Talk Frame Model - Mathematical Argument
Limits - continuity

Think

Consider the statements along with the graph below:

a.) $\lim_{x\to 0} f(x) = 1$ b.) $\lim_{x\to 1} f(x) = 1$

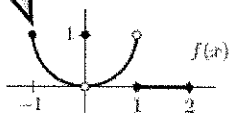
Create a mathematical argument to justify whether each one is TRUE or FALSE.

Talk Idea

Talk Idea

Talk Idea

Anticipated ideas



- Limit existence from one-side or both sides
- Limit existing vs. function existing
- Concept of approaching a value

What's a potential goal(s) for the discussion? What should students come to better "understand"?

We Understand

- Limit theory has many different perspectives
- Behavior of functions need to be justified mathematically
- Introduction to Continuity Theory

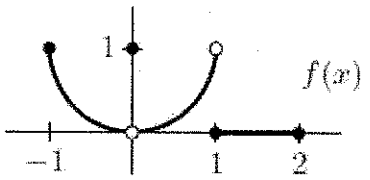
Name_____ Date_____ Period_____

Talk Frame Discussion

Consider the statements along with the graph below:

a.) $\lim_{x \to 0} f(x) = 1$

b.) $\lim_{x \to 1} f(x) = 1$



Create a mathematical argument to justify whether each one is TRUE or FALSE.

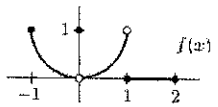
Name Group A Date _____ Period _____

AP Calculus

Talk Frame Discussion

Consider the statements along with the graph below:

a.) $\lim_{x \to 0} f(x) = 1$ b.) $\lim_{x \to 1} f(x) = 1$



Create a mathematical argument to justify whether each one is TRUE or FALSE.

a.) TRUE: The function value is 1 at $x=0$, even though the values around it approach 0.

b.) FALSE: The function value is at 0 not at 1 as shown by the hole in the graph as x approaches 1. Therefore the limit is 0.

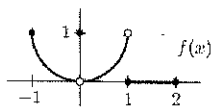
Name Group B Date _____ Period _____

AP Calculus

Talk Frame Discussion

Consider the statements along with the graph below:

a.) $\lim_{x \to 0} f(x) = 1$ b.) $\lim_{x \to 1} f(x) = 1$



Create a mathematical argument to justify whether each one is TRUE or FALSE.

a.) FALSE: Even though the function value is at 1, the other values approach 0 as $x \to 0$ so the limit is 0.

b.) FALSE: The limits from either side as $x \to 1$ are going to different values so the limit can't exist there.

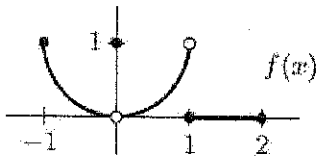
Name Group C Date _____ Period _____

AP Calculus

Talk Frame Discussion

Consider the statements along with the graph below:

a.) $\lim_{x\to 0} f(x) = 1$ b.) $\lim_{x\to 1} f(x) = 1$



Create a mathematical argument to justify whether each one is TRUE or FALSE.

a.) FALSE: The function values around $x=0$ are close to 0 so that is the limit. It doesn't matter where $f(0)$ really is.

b.) TRUE: The function values are heading to 1 as we get close to $x=1$. It doesn't matter that $f(1)=0$ because we just approach.