Module 1: What is Argumentation?

Opening Activities: Getting to Know Our Group

Establishing Community Agreement

Guiding Question: **What are some of the things that are important for a group to agree to in order for that group to work well together?**

1: Think (2-3 min.)

Think about 2 or 3 things that are necessary for a group to work well together and jot them down on a piece of paper.

2: Share (2-3min.)

In small groups, share your thoughts with your group members and decide which you feel are important enough to propose to the whole group for inclusion into our community agreement.

3: Review

Review and agree on proposals. Understand that we may make revisions and/or additions that might be necessary.

4: Write and Post (and Revisit)

The reporter/recorder will write them on a sentence strip and post on the wall to make them visible and public.

Argumentation

Mathematical *argumentation* involves a host of different "thinking" activities: generating conjectures, testing examples, representing ideas, changing representation, trying to find a counterexample, looking for patterns, etc.

Standards of Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. Reason abstractly and quantitatively.
3. **Construct viable arguments and critique the reasoning of others.**
4. Model with mathematics.
5. Use appropriate tools strategically.
6. **Attend to precision.**
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

How do mathematical practices relate to student reasoning?

- **CCCSM MP 3 – Construct viable arguments and critique the reasoning of others.** The practice begins, "Mathematically proficient students ***understand and use stated assumptions, definitions, and previously established results in constructing arguments …"***

Thinking is the hardest work there is, which is probably the reason why so few engage in it. - Henry Ford

Teachers Will Be Able To:

- ✓ Develop a deeper understanding of argumentation and its potential in the math classroom.
- ✓ Analyze mathematical arguments within the three components of an argument.

Overarching Guiding Questions:

- What is a mathematical argument? What "counts" as an argument?
- What is the purpose(s) of argumentation in mathematics? In the math classroom?
- What does student argumentation look like at different levels of proficiency?

A Mathematical Argument

- A Mathematical Argument <u>is…</u>
  - A sequence of statements and reasons given with the aim of demonstrating that a claim is true or false

- A Mathematical Argument <u>is not</u>…
  - An explanation of what you did (steps)
  - A recounting of your problem solving process
  - Explaining why you personally think something is true for reasons that are not necessarily mathematical (e.g., popular consensus; external authority, intuition, etc. *It's true because John said it, and he's always always right.*)

Let's take a look…

Abbott & Costello: 7 x 13 = 28
https://www.youtube.com/watch?v=xkbQDEXJy2k

Your Turn...

When you add any two consecutive numbers, the answer is always odd.

Is this statement true or false?

Write a mathematical argument to support your claim.

Your Turn…

When you add any two consecutive numbers, the answer is always odd.

Share your arguments with your group.
What similarities and differences do you notice?

Structure of an Argument

WHAT?

What is your response?

WHY? / HOW?

Why is this true? How do you know this is true?

What mathematical WORK related to the problem will support your response?

What mathematical RULES will support your work and your response?

Analyzing Student Arguments on the Consecutive Sums Task

You have 4 sample student responses to the Consecutive Sums Task.

For each student argument:

(1) Discuss the student's argument.
(2) Determine if the argument shows the claim is true.

When you add any two consecutive numbers, the answer is always odd.

Micah's Response

5 and 6 are consecutive numbers, and 5 + 6 = 11 and 11 is an odd number.

12 and 13 are consecutive numbers, and 12 + 13 = 25 and 25 is an odd number.

1240 and 1241 are consecutive numbers, and 1240 +1241 = 2481 and 2481 is an odd number.

That's how I know that no matter what two consecutive numbers you add, the answer will always be an odd number.

Example – Analyzing the Structure

Micah's Response

5 and 6 are consecutive numbers, and 5 + 6 = 11 and 11 is an odd number.

12 and 13 are consecutive numbers, and 12 + 13 = 25 and 25 is an odd number.

1240 and 1241 are consecutive numbers, and 1240 +1241 = 2481 and 2481 is an odd number.

That's how I know that <u>no matter what two consecutive numbers you add, the answer will always be an odd number.</u>

Evidence
3 examples that fit the criterion

Warrant
(implicit) because if it works for 3 of them, it will work for all

Claim

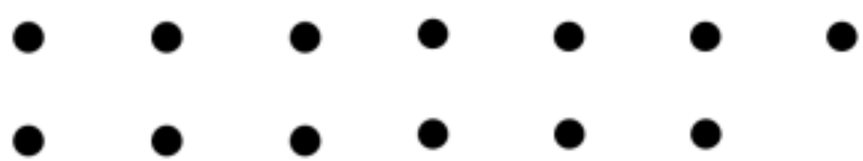
When you add any two consecutive numbers, the answer is always odd.

Roland's Response

The answer is always odd.

A number + The next number =

An odd number



There's always one left over when you put them together, so it's odd.

When you add any two consecutive numbers, the answer is always odd.

Angel's Response

Consecutive numbers go even, odd, even, odd, and so on. So if you take any two consecutive numbers, you will always get one even and one odd number.

And we know that when you add any even number with any odd number the answer is always odd.

That's how I know that no matter what two consecutive numbers you add, the answer will always be an odd number.

When you add any two consecutive numbers, the answer is always odd.

<u>Kira's Response</u>

Consecutive numbers are n and n+1.

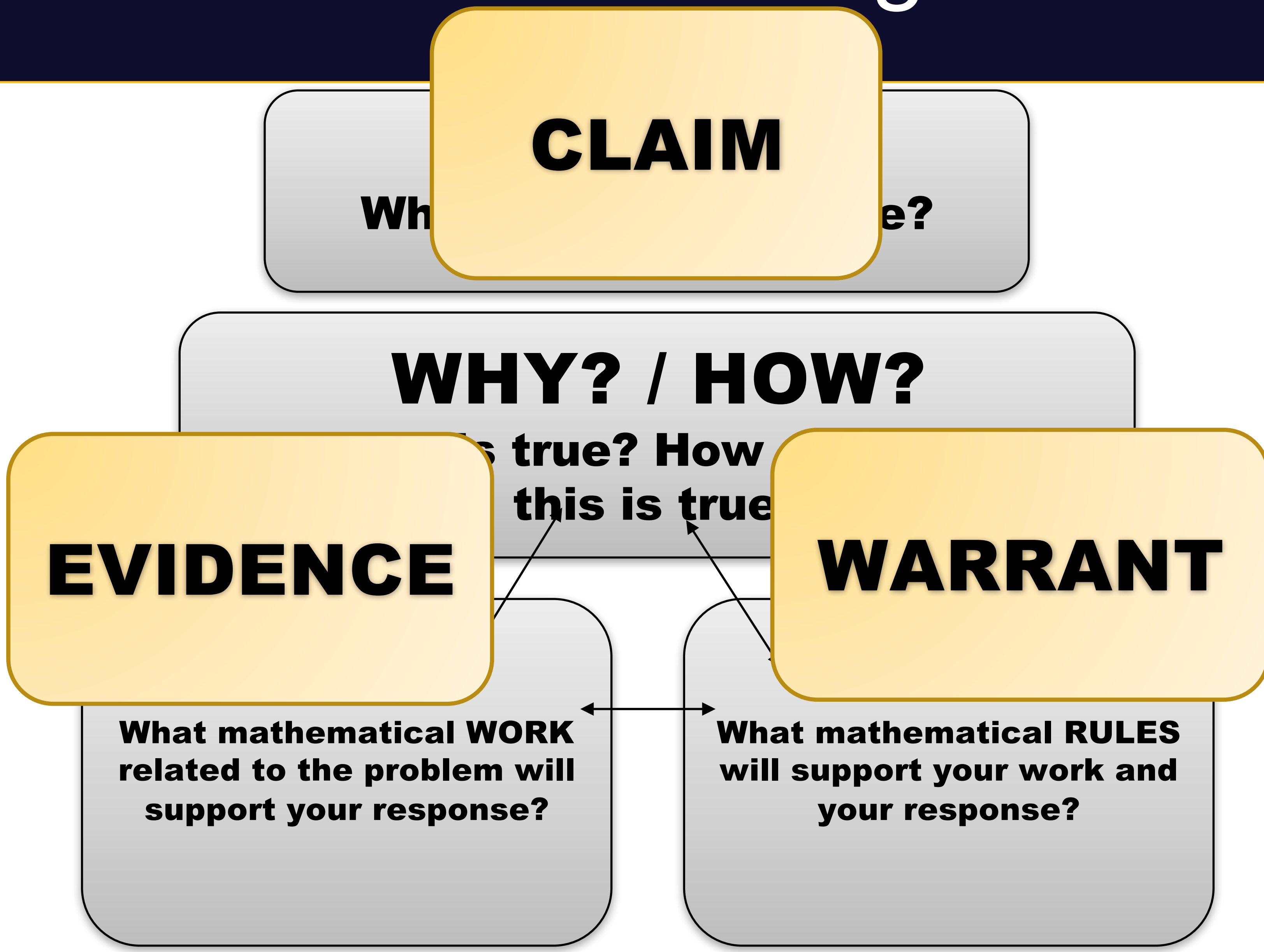
Add the two numbers:
$n + (n+1) = 2n + 1$

You get $2n + 1$ which is always an odd number, because an odd number leaves a remainder of 1 when divided by 2. (2 goes into $2n + 1$ n times, with a remainder of 1)

Comments on the approaches

- Example based (Micah)
  - May not be enough to PROVE a claim
- Narrative (Angel)
  - Stated warrant may also require justification
- Pictorial (Roland)
  - Can be strong evidence but insufficient warrant
- Symbolic (Kira)
  - Can be strong evidence but insufficient warrant

Structure of an Argument



Structure of an Argument

CLAIM WHAT?

Your answer, result, or solution

What you believe to be true (or false)

Your stance or position to be supported

EVIDENCE

WHY / HOW?

Math work that can help support your claim

Evidence can take the form of equations, graphs, tables, diagrams, computations, and even words

WARRANT

WHY / HOW?

Math rules that can help support your claim

Warrants are often general, applying to many situations; warrants can be definitions, previously proven theorems, or other established truths

Analyzing Student Arguments on the Consecutive Sums Task

- Work through one of the student work samples together with your group
  - Use the CLAIMS, WARRANTS, EVIDENCE vocabulary
- Think about the strengths and weaknesses of each argument.
- Mark on the student work samples handouts
  - Highlight and make notes about what people notice.

Reflection Questions

How does your argument compare to the student samples?

How would you modify your work to make a stronger argument?

Bridging to Practice

Bridging to Practice

Protocol-Guided Student Work Sample Sorting Activity

Materials: Protocol and Set of Student Work Samples

Process:

- Individual sort
- Group discussion and group sort
- Full group discussion/debrief

Purposes: protocols help organize professional conversations

- help maintain focus on shared goals (for depth); provide time limits to ensure conversation covers required ground (for breadth and completeness)
- encourage collaboration, active listening, and respectful dialogue that includes differences of opinions

Closure

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