HALVES, THIRDS, SIXTHS PROBLEM

STUDENT WORK SAMPLE ARGUMENTATION RESOURCE PACKET

This packet was produced as part of the Bridging Math Practices Math-Science Partnership Grant (2014 -2015).

The purpose of the packet is to help a) reveal what students can do with respect to generating an argument in response to mathematical questions, including the variety of their arguments; b) highlight features that should be considered when reviewing students' arguments, and c) identify what counts as a *quality* argument in light of the review criteria.

What is a mathematical argument?

A mathematical argument is

a sequence of statements and reasons given with the aim of demonstrating that a claim is true or false.

This links to the Connecticut Core Standards of Mathematical Practice #3, *construct viable arguments and critique the reasoning of others,* as well as other standards.

This resource packet is a product of work by participants in the UConn Bridging Math Practices Math-Science Partnership Grant, which included faculty and graduate students from the University of Connecticut's Neag School of Education and Department of Mathematics, and teachers and coaches from the Manchester Public Schools, Mansfield Public Schools, and Hartford Public Schools. This resource packet reflects significant contributions from Jeff Burnham, Michael DiCicco, Jocelyn Dunnack, Kelly Haggerty, Catherine Hain, Karen Herrick, Brenda Moulton, Charles Warinsky, and Patrice Welch. Many thanks for all their insights and contributions! For more information about the grant, or for additional argumentation-related materials and resource, please see the project website:
http://bridges.uconn.education.edu
The Mathematics and Science Partnership (MSP) grant is a federal program funded under Title II, Part B, of the *Elementary and Secondary Education Act* and administered by the U.S. Department of Education.

What is a high quality mathematical argument?

A high quality mathematical argument is an argument that shows that a claim must be true. It leaves little room to question. The chain of logic leads the reader to conclude that the author's claim is true.

What are the characteristics of a high quality argument? A high quality argument can be described by the following components and criteria:

Criteria	**Description**
1. A clearly stated **claim**	The claim is what is to be shown true or not true.
2. The necessary **evidence** to support the claim	Evidence can take the form of equations, tables, charts, diagrams, graphs, words, symbols, etc. It is one's "work" which provides the information to show something is true/false.
3. The necessary **warrants** to connect the evidence to the claim	Warrants can take the form of definitions, theorems, logical inferences, agreed upon facts. Warrants explain how the evidence is relevant for the claim, and collectively they chain the evidence together to show the claim is true or false.
4. **Language use and computations** are at a sufficient level of precision and accuracy	The language used and computations must be at a sufficient level of precision or accuracy to support the argument. Language use needs to be precise enough to communicate the ideas with sufficient clarity.

These criteria are helpful for discussions. It is important not to lose sight of the "big picture" however, and that is whether the argument offered shows that the claim is (or is not) true. This is the goal and purpose of a mathematical argument. You will see in many of these packets that students can approach an argumentation prompt from many different perspectives. It matters less *which* mathematical tools they use, and matters more whether their chain of reasoning compels the result.

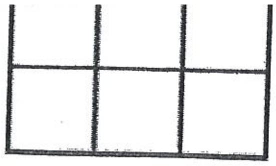
In this packet you will find

1. A blank copy of the task: ‘Halves, thirds and sixths’ and a description of the task implementation and/or other important considerations regarding student work samples included in this packet.
2. A protocol that can help you and your colleagues discuss student work related to this task.
3. Selected work samples on this task from 3rd, 4th, 5th and 6th grade students in classes of teacher participants in the UConn Bridging Math Practices project to be used with the protocol.
4. Work Samples Classification and Commentaries: the student work samples ordered by whether they seem to be *high, adequate, or low quality* responses with respect to the criteria described on the previous page; along with commentaries that support the classification. Among the samples are some that present a well-structured argument, but have important mathematical flaws, which prevent them from being classified as the highest quality.

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much “weight” people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional *discussions* and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

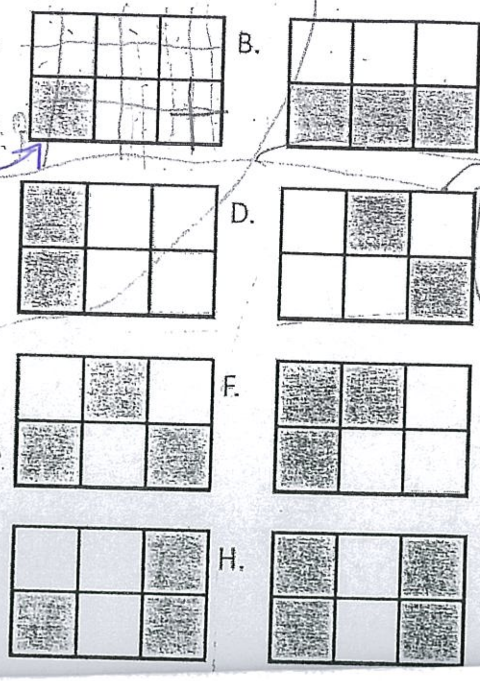
CONTEXT
This argumentation resource packet was developed as a collaborative effort across grades 3 through 6 teachers to learn about how students' arguments may change across grade levels. The same task was given to all students except for the grade 6 task to meet their students' learning development.

Because this task was done across grades, we have two different ways you can look at the samples. You may look at only the samples for a given grade level. You might also want to look at the "AllGrades" packet of student work samples which has 3-4 pieces of student work per grade 3 – 6. You might also choose to just look at the samples for one grade level.

2  $2 \times 3 = 6$

There is six partin the rectangle

b. What fraction of the area of each rectangle is shaded? Name the fraction in as many ways as you can. Explain your answers.



$\frac{8}{48}$ $\frac{4}{24}$ $\frac{2}{12}$ $\frac{1}{6}$

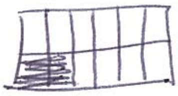
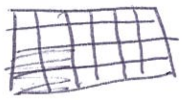
$\frac{16}{96}$ $\frac{8}{48}$ $\frac{4}{24}$ $\frac{2}{12}$ $\frac{1}{6}$ $\frac{1}{3}$ ... $\frac{1}{3}$

$\frac{1}{2}$ You can't put it 24 because you can't split 6 into 4 equal ... $\frac{2}{6}$ $\frac{4}{12}$ $\frac{8}{24}$ $\frac{}{48}$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

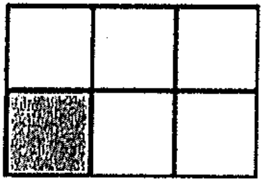
$\frac{2}{3}$ $\frac{4}{6}$ $\frac{8}{12}$ $\frac{16}{24}$

Student:

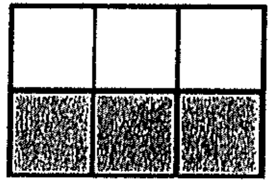
"I took the array in letter A  which is $\frac{1}{6}$, and broke it into 12 smaller equal parts -  which shows that $\frac{1}{6}$ and $\frac{2}{12}$ take up the same part of the whole. I can divide it into 24 equal parts  and now it shows that $\frac{1}{6} = \frac{2}{12}$ which is also equal to $\frac{4}{24}$. I noticed a pattern. Each step the numerator doubles + the denominator doubles."

Student 2

1/6 A.

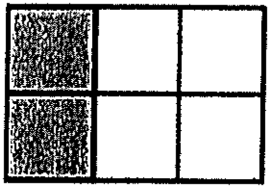


B.

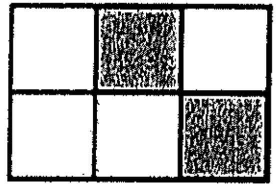


$\frac{3}{6}$

$\frac{4}{6}$ $\frac{2}{6}$ C.

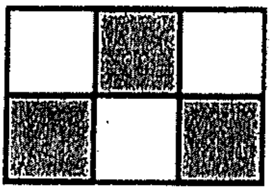


D.

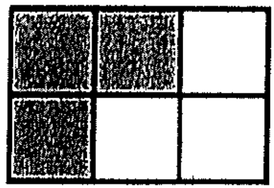


$\frac{2}{6}$ $\frac{4}{6}$

$\frac{3}{6}$ E.

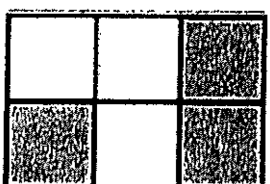


F.

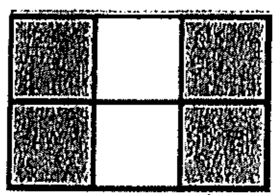


$\frac{3}{6}$

$\frac{3}{6}$ G.



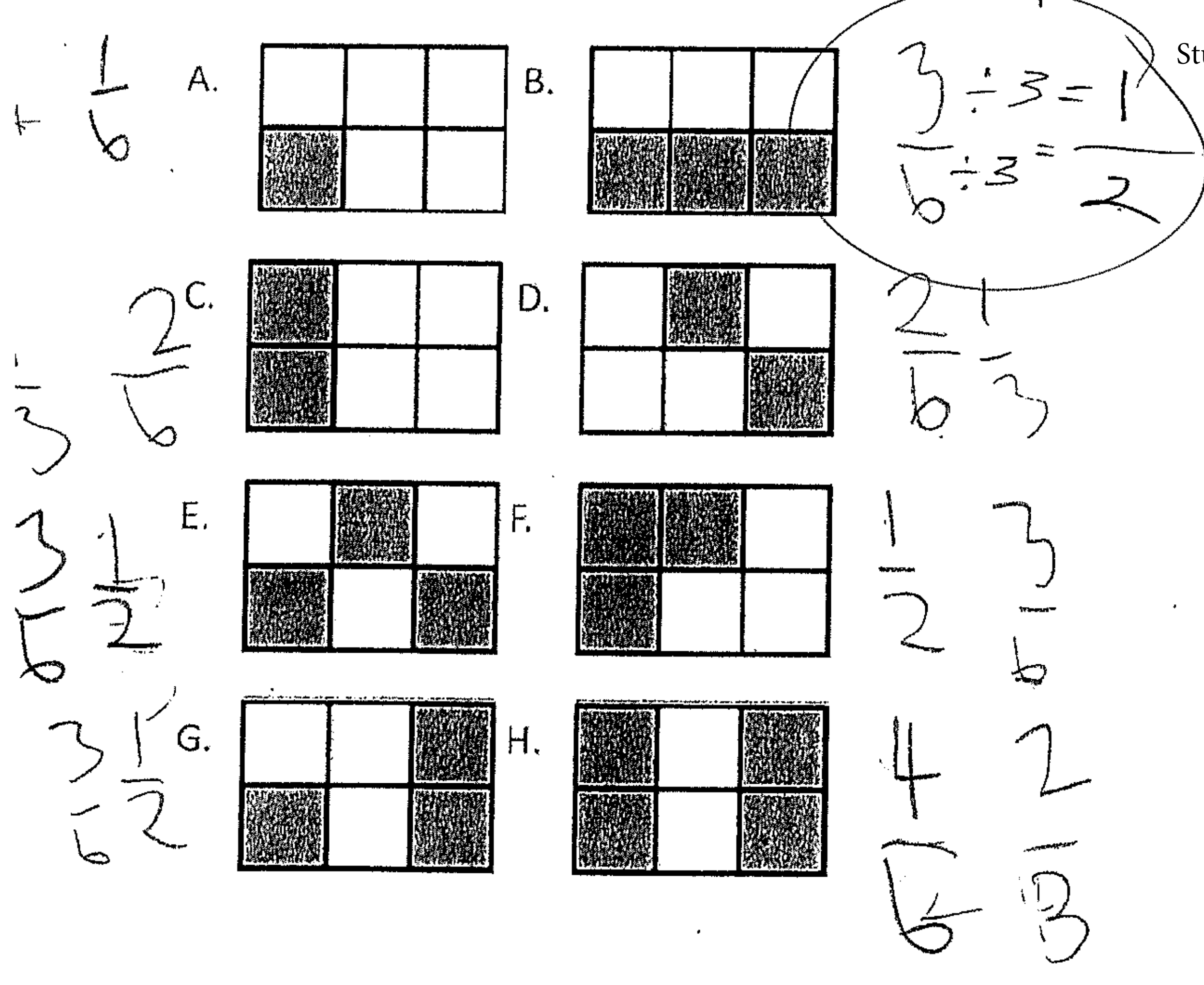
H.



$\frac{4}{6}$ $\frac{2}{2}$

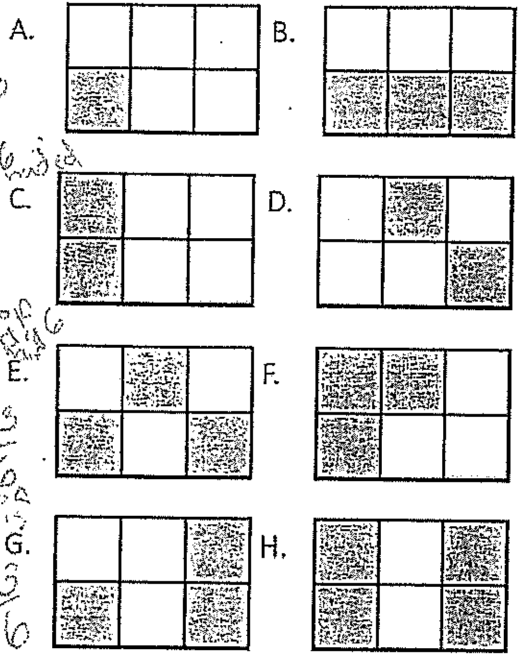
4 Parts are Blue,

Student 3



What fraction of the area of each rectangle is shaded? Name the fraction in as many ways as you can. Explain your answers.

Student 4



A. $\frac{1}{6}$ because 1 out of 6 is shaded

B. $\frac{3}{6}$ or $\frac{1}{2}$ because 3 out of 6 is shaded 3 is half of 6

C. $\frac{2}{6}$ or $\frac{1}{3}$ because 2 out of 6 is shaded

D. $\frac{2}{6}$ or $\frac{1}{3}$ because 2 out of 6 is shaded and because 2 is a third of 6

E. $\frac{3}{6}$ or $\frac{1}{2}$ because 3 out of 6 is shaded 3 is half of 6

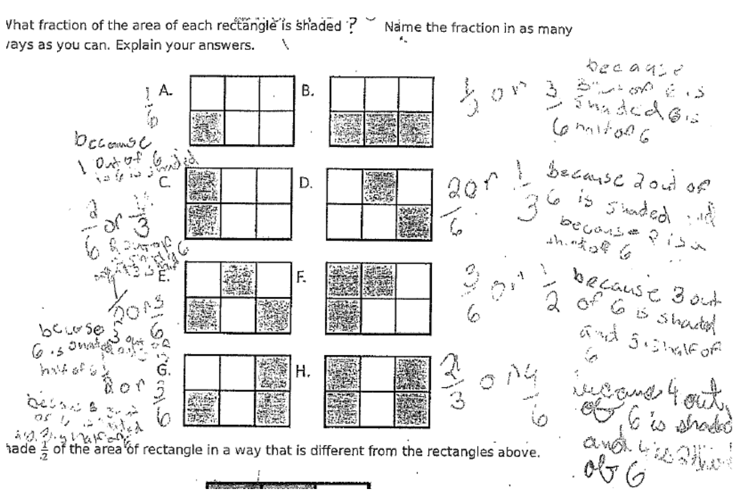
F. $\frac{3}{6}$ or $\frac{1}{2}$ because 3 out of 6 is shaded and 3 is half of 6

G. $\frac{3}{6}$ or $\frac{1}{2}$ because 3 out of 6 is shaded and 3 is half of 6

H. $\frac{4}{6}$ or $\frac{2}{3}$ because 4 out of 6 is shaded and 4 is 2 thirds of 6

Shade $\frac{1}{2}$ of the area of rectangle in a way that is different from the rectangles above.

Student A



Commentary

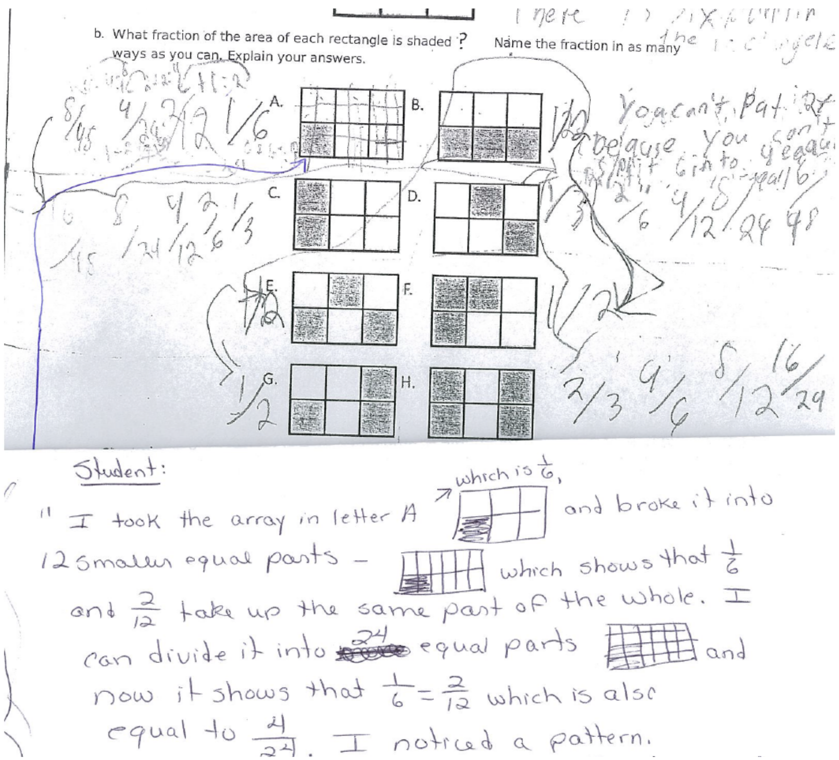
This student's argument was categorized as **high quality**.

Student A claims that 2/6 and 1/3 are equivalent fractions. Student A also claims that 2/3 or 4/6 are equivalent fractions. Student A states the that "because 2 out of 6 is shaded and because 2 is 1/3 of 6."(D.) He or she also states that "because 4 out of 6 is shaded and 4 is 2/3 of 6."(H.) Student A demonstrates an implied understanding of inverse operations of multiplication and division by a whole to compute equivalent fractions. There could be a judgment call in the implied mathematical computation of multiplying and dividing by a whole, as is suggested through the explanations.

Argumentation Components

Claim	**Evidence**
The claim is stated as the equivalent fractions in each case. For example, in D. that 2/6 and 1/3 are equivalent fractions	Equivalent fractions are stated for each model
Warrants	**Language & Computation**
The student states they are equivalent because they name the shaded part of fraction shown. For example, in H. they state 4 is 2/3 of 6.	All mathematical computations and statements are correct.

Student B



Commentary

This student's argument was categorized as **high quality**.

Student B demonstrates partitioning of a whole (same whole) to create equivalent fractions. The student shows that by partitioning (see A), she is creating equal parts of the same whole and is able to list numerical equivalent fractions that match an array model of the fractions as well. Only picture A shows this use of partitioning and is assumed for the other fractions.

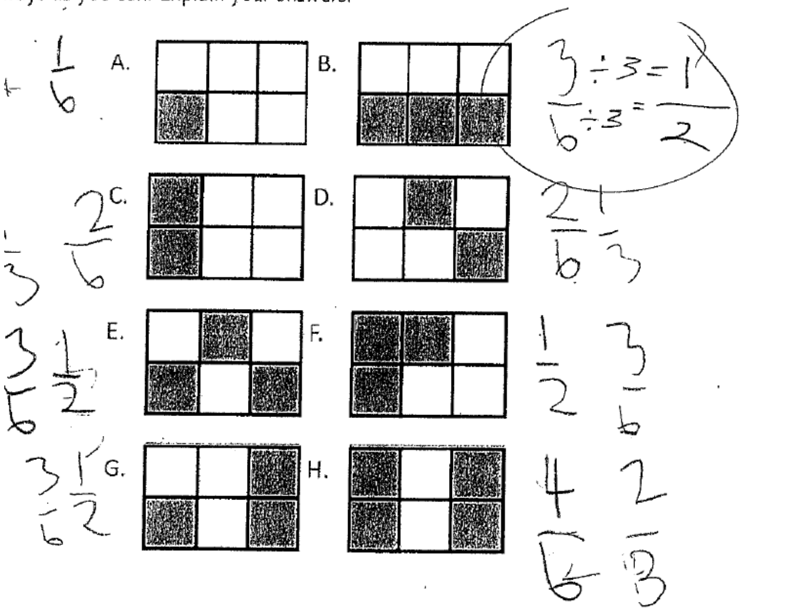
The student's written explanation clearly demonstrates an understanding of equal parts of a whole and correctly supports the claim.

Argumentation Components

Claim	Evidence
8/48, 4/24, 2/12, 1/6 are all equivalent fractions.	The picture in A shows different partitioning of a whole that were used to generate the lists of equivalent fractions.
Warrants	**Language & Computation**
The explanation below the figure provides a strong connection between the visual evidence and the claim. Example of warrants offered: "1/6 and 2/12 take up the same part of the whole."	All mathematical computations and statements are correct.

Student C

What fraction of the area of each rectangle is shaded blue? Name the fraction in as many ways as you can. Explain your answers.



Commentary

This student's argument was categorized as **adequate quality**.

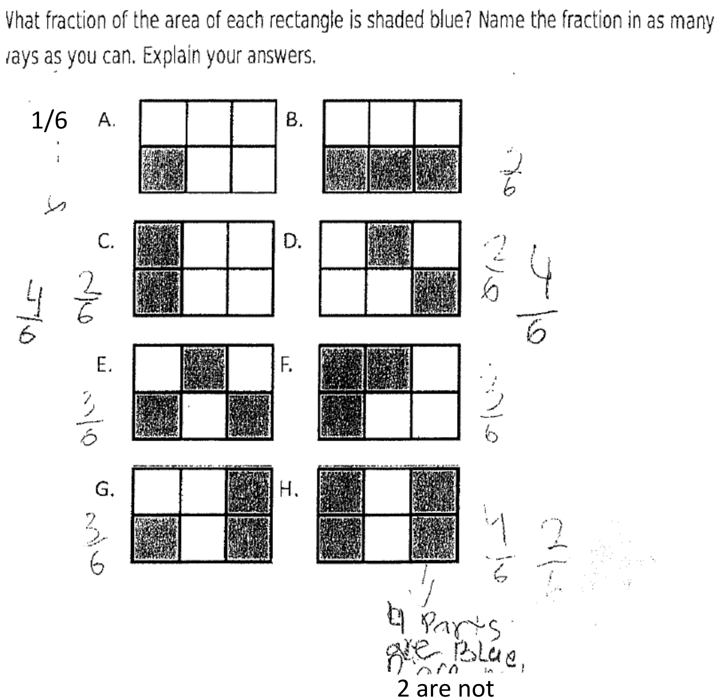
Student C showed equivalent fractions through dividing both numerator and denominator by the same whole number; however there is no rationale or warrant for why this generates an equivalent fraction. Student C only provided one example as evidence.

The argument could be strengthened by explicitly stating that 3/3 is a form of 1, which would give an equivalent fraction.

Argumentation Components

Claim	**Evidence**
Student correctly names one equivalent fraction for each model.	See student work on part B.
Warrants	**Language & Computation**
Warrants are missing.	All mathematical computations and statements are correct.

Student D



Commentary

This student's argument was categorized as **low quality**.

The student explicitly states that 4 parts are blue and 2 are not, which explains how 4/6 was obtained. However, *the work does not display understanding of equivalent fractions. The student is simply naming the shaded and un-shaded regions in each rectangle without addressing the part of the prompt about different fractions that represent the shaded region.*

The work might indicate a misunderstanding between naming fractions in different ways (equivalent fractions) and naming all fractions represented in the picture (definition of fractions).

Argumentation Components

Claim	Evidence
That the shaded part or number over a whole is a fraction.	Student identified and labeled fractions as parts of a whole.
Warrants	**Language & Computation**
Warrants are missing.	The fractions are correct; although they do not completely address the prompt in the task. Very little language is used; but what *is* stated contains no errors.

Key Connecting Sorting Packet to Argumentation Resource Packet

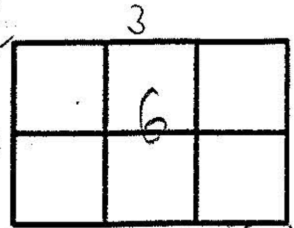
Student number (Soring Packet)	Resource Packet Sample
1	B
2	D
3	C
4	A
5	
6	
7	
8	
9	

Student number (Soring Packet)	Resource Packet Sample (category)
4	A (high)
1	B (high)
3	C (adequate)
2	D (low)
	E ( )
	F ( )
	G ( )
	H ( )
	I ( )

a. A small square [redacted]tangle? Explain. 4th gr

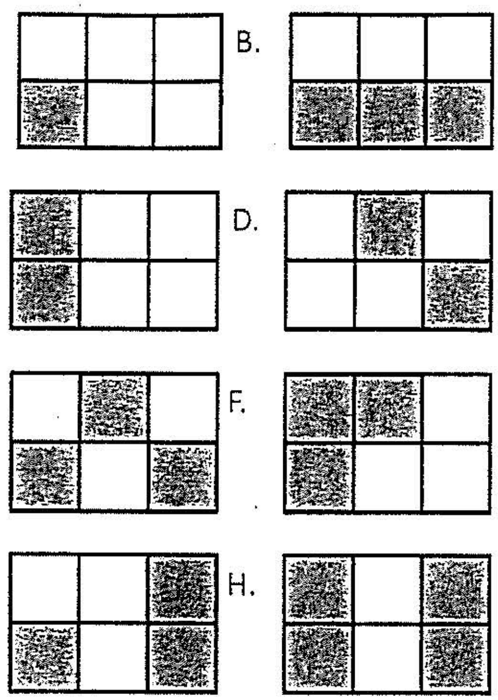
Student 1

T if you multiple 3×2 you will get the area. if you 2 multiplie 2 and 2 and 2 and 3 you will get perimet,



$$\begin{array}{r} 2 \\ \times 3 \\ \hline 6 \end{array}$$

b. What fraction of the area of each rectangle is shaded blue? Name the fraction in as many ways as you can. Explain your answers.



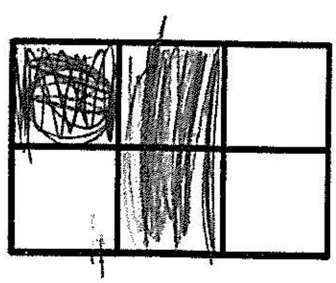
A. $\frac{1}{6}$ B. $\frac{1}{2}$

C. $\frac{2}{6}$ D. $\frac{2}{6}$

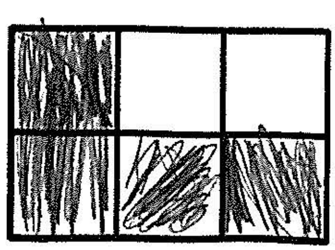
E. $\frac{1}{2}$ F. $\frac{1}{2}$

G. $\frac{1}{2}$ H. $\frac{4}{6}$

c. Shade $\frac{1}{2}$ of the area of rectangle in a way that is different from the rectangles above.

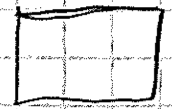


d. Shade $\frac{2}{3}$ of the area of the rectangle in a way that is different from the rectangles above.



6/12/15

Student 1

a. 

~~$2 * 2$~~ ~~$3 + 3$~~

~~$2 * 3 = 6$~~

$$\begin{array}{r} 2 \\ \times\ 3 \\ \hline 6 \end{array}$$

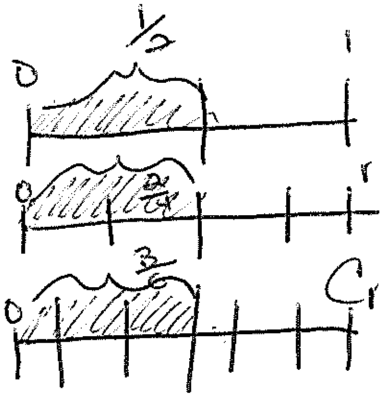
b. for the fraction A-H you would the part I shadid in ex A it's $\frac{1}{6}$ only one is shadid in and you would cont the rast.

Student 2

A: 1/6, 2/12, 3/8

1 2 3

B: $\frac{1}{2}$, 4, 6



with models for each

C: $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$

with models for each

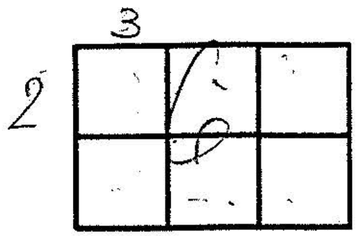
I know these fractions are equivalent because the shaded ~~part~~ area for each equivalent fraction is the same (amount).

> models demonstrate understands of comparison of equivalent wholes. Clearly labeled models

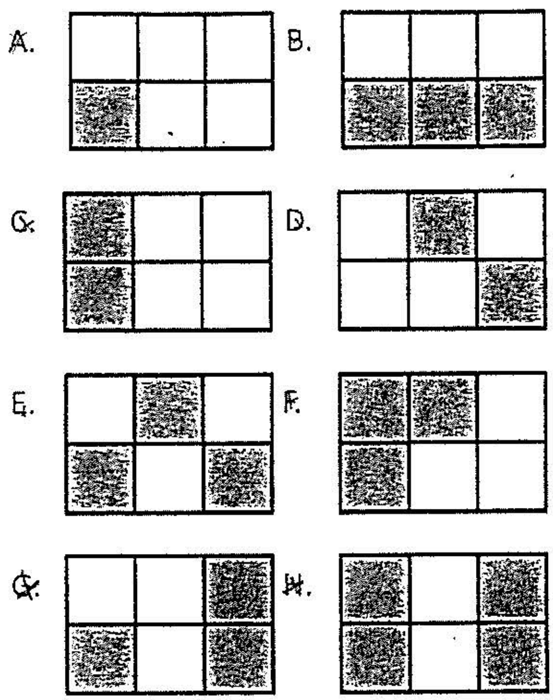
[redacted] the area of this rectangle? Explain.

4th gr June 12, 2012

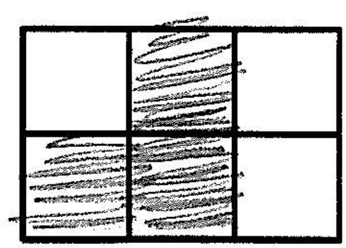
Student 3



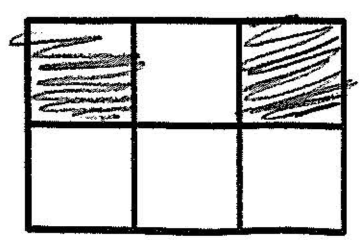
b. What fraction of the area of each rectangle is shaded blue? Name the fraction in as many ways as you can. Explain your answers.



c. Shade $\frac{1}{2}$ of the area of rectangle in a way that is different from the rectangles above.



d. Shade $\frac{2}{3}$ of the area of the rectangle in a way that is different from the rectangles above.



a. $2 \times 3 = 6$. The formula for area is $L \times W = A$

bA. $\frac{1}{6}$ $\frac{2}{12}$ $\frac{4}{24}$ $\frac{8}{48}$ $\frac{16}{96}$ $\frac{32}{192}$ $\frac{64}{384}$ $\frac{128}{768}$ $\frac{256}{1536}$ each time I make the fraction smaller, but all of the fractions listed above are equal

bB. $\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$ $\frac{5}{10}$ $\frac{6}{12}$ $\frac{7}{14}$ $\frac{8}{16}$ $\frac{9}{18}$ $\frac{10}{20}$ $\frac{11}{22}$ $\frac{12}{24}$ $\frac{13}{26}$ $\frac{14}{28}$ $\frac{15}{30}$ $\frac{16}{32}$ $\frac{17}{34}$ $\frac{18}{36}$ (I can keep going but that would take a whille.) all of these fractions are equal because they are halves.

bC $\frac{1}{3}$ $\frac{2}{6}$ $\frac{3}{12}$ $\frac{4}{24}$ $\frac{5}{48}$ $\frac{6}{96}$ $\frac{7}{192}$ $\frac{8}{384}$ $\frac{9}{768}$ $\frac{10}{1536}$ all of these are equal, and they all can be reduced to 3rd's (exept for the $\frac{1}{3}$).

bD. $\frac{1}{3}$ $\frac{2}{6}$ $\frac{4}{12}$ $\frac{8}{24}$ $\frac{16}{48}$ $\frac{32}{96}$ $\frac{64}{192}$ $\frac{128}{384}$ $\frac{256}{768}$ $\frac{512}{1536}$ These are all equal and cam be reduced to $\frac{1}{3}$ (exept for the $\frac{1}{3}$).

bE. $\frac{1}{2}$ - $\frac{18}{36}$ all of the fractions are halves.

bF $\frac{1}{2}$ - $\frac{18}{36}$ all of the fractions are halves.

bG. $\frac{1}{2}$ _ $\frac{18}{36}$ all of the fractions are equal because they are halves

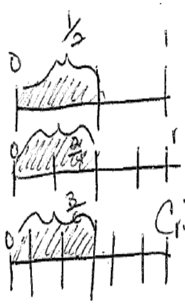
bH. $\frac{4}{6}$ $\frac{8}{12}$ $\frac{16}{24}$ $\frac{32}{48}$ $\frac{64}{96}$ $\frac{128}{192}$ all of these fractions are equal because if reduced, all can come to $\frac{4}{6}$

This will not be in the final packet.
This is for our records here.

- Task title: Halves, Thirds and Sixths
- Grade level of task: 4th
- Team members' names: Charles Warinsky and Catherine Hain

Student A

A: 1/6, 2/12, 3/18
B: 1/2, 2/4, 3/6



with models for each

C: $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$

with models for each

I know these fractions are equivalent because the shaded ~~parts~~ area for each equivalent fraction is the same (amount).

Commentary

This student's argument was categorized **High Quality**.

Student A's claim is that the fractions they wrote were equivalent to the fraction represented in the rectangle.

Student A provided clearly labeled models (using area and number lines) as evidence and explained why the models show that the fractions are equivalent.

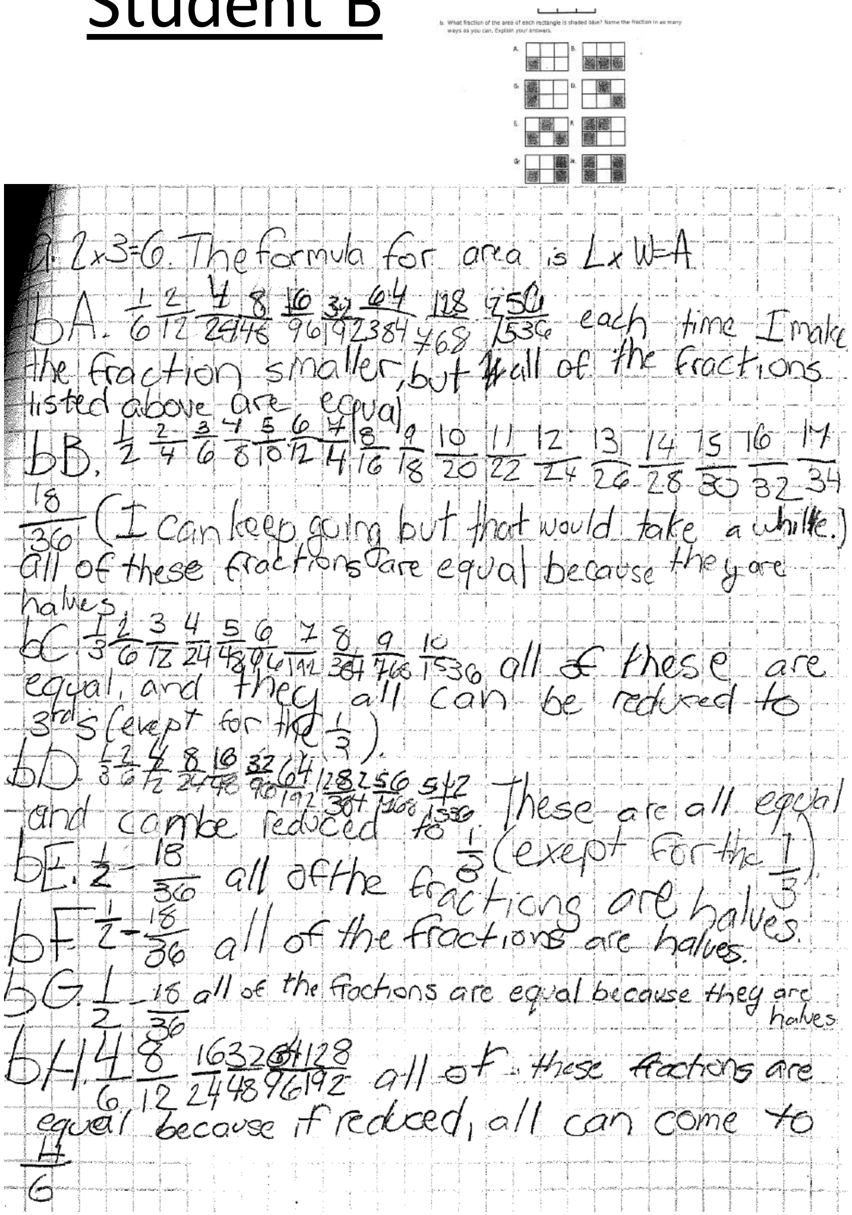
Student A correctly named at least two equivalent fractions for the given fraction and drew models that represented how all of the fractions show the same area or value.

Models may include rectangles or number lines and should clearly demonstrate understanding of comparison of equivalent wholes.

Argumentation Components

Claim	Evidence
I know these are fractions equivalent.	Sufficient examples of equivalent fractions are given using area models and number lines.
Warrants	**Language & Computation**
The warrant states "the shaded area for each equivalent fraction is the same (amount)."	The mathematical language used is precise and ideas flow clearly. Vocabulary used includes: -equivalent -equivalent fraction -same amount

Student B



a. $2 \times 3 = 6$. The formula for area is $L \times W = A$

bA. $\frac{1}{6}$ $\frac{2}{12}$ $\frac{4}{24}$ $\frac{8}{48}$ $\frac{16}{96}$ $\frac{32}{192}$ $\frac{64}{384}$ $\frac{128}{768}$ $\frac{256}{1536}$ each time I make the fraction smaller, but all of the fractions listed above are equal.

bB. $\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$ $\frac{5}{10}$ $\frac{6}{12}$ $\frac{7}{14}$ $\frac{8}{16}$ $\frac{9}{18}$ $\frac{10}{20}$ $\frac{11}{22}$ $\frac{12}{24}$ $\frac{13}{26}$ $\frac{14}{28}$ $\frac{15}{30}$ $\frac{16}{32}$ $\frac{17}{34}$ $\frac{18}{36}$ (I can keep going but that would take a while.) all of these fractions are equal because they are halves.

bC $\frac{1}{3}$ $\frac{2}{6}$ $\frac{3}{12}$ $\frac{4}{24}$ $\frac{5}{48}$ $\frac{6}{96}$ $\frac{7}{192}$ $\frac{8}{384}$ $\frac{9}{768}$ $\frac{10}{1536}$ all of these are equal, and they all can be reduced to 3rds (exept for the $\frac{1}{3}$).

bD $\frac{1}{3}$ $\frac{2}{6}$ $\frac{4}{12}$ $\frac{8}{24}$ $\frac{16}{48}$ $\frac{32}{96}$ $\frac{64}{192}$ $\frac{128}{384}$ $\frac{256}{768}$ $\frac{512}{1536}$ These are all equal and can be reduced to $\frac{1}{3}$ (exept for the $\frac{1}{3}$)

bE. $\frac{1}{2}$ - $\frac{18}{36}$ all of the fractions are halves.

bF $\frac{1}{2}$ - $\frac{18}{36}$ all of the fractions are halves.

bG. $\frac{1}{2}$ - $\frac{18}{36}$ all of the fractions are equal because they are halves

bH. $\frac{4}{6}$ $\frac{8}{12}$ $\frac{16}{24}$ $\frac{32}{48}$ $\frac{64}{96}$ $\frac{128}{192}$ all of these fractions are equal because if reduced, all can come to $\frac{4}{6}$

Commentary

This student's argument was categorized as ***Adequate quality****.*

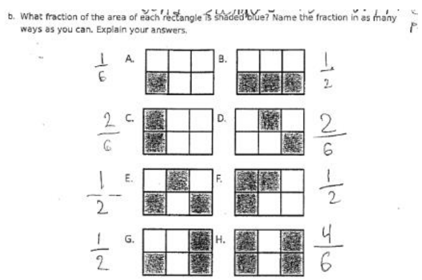
Student B's claim is that the fractions are equivalent. Student B provided multiple examples of equivalent fractions and evidence of how the student found some of these examples, as in example bC, bD and bH, yet the warrants are incomplete. There is not enough explanation of why the fractions are equivalent other than the statement that they can be reduced to the same simplest form.

There is also a misconception about making a fraction "smaller" versus reducing or simplifying it.

Argumentation Components

Claim	Evidence
The fractions I listed are equal.	*Sufficient examples are provided.*
Warrants	**Language & Computation**
Warrants are incomplete: "All fractions can be reduced to (simplest form)."	*The mathematical language used is precise and ideas flow clearly. Vocabulary used includes:* *-reduced* *-equal*

Student C



b. for the fraction A-H you would the part I shadid in ex A it's $\frac{1}{6}$ only one is shadid in and you would cont the rast.

Commentary

This student's argument was categorized as **Low quality**.

Student C identified the shaded portions of the rectangles but did not create equivalent fractions. There is no claim, warrant or examples.

Argumentation Components

Claim	Evidence
None	None
Warrants	**Language & Computation**
None	None

Rubric

Category	Description with Examples/Non-Examples	0	1	2	3
1. The **claim** presents the position being taken.	The claim is what is to be shown true or not true. It may be explicitly stated or implied through examples. *Example:* ½, 2/4, 3/6, 4/8 (implied); 2/4, 3/6, and 4/8 are equivalent to 1/2 *Non-example:* ½ = 4/6; not equivalent fractions	No claim	Claim is included but not clear	Claim is clearly articulated	---
2. **Evidence** supports the claim.	Evidence can take the form of equations, tables, charts, diagrams, graphs, words, symbols, etc. It is one's "work" which provides the information to show something is true/false. *Example:* 1/2, 2/4, 3/6, 4/8, etc. *Non-example:* incorrect statements about equivalent fractions	No evidence	Minimal evidence is included, <u>or</u> evidence is unrelated to the claim, <u>or</u> major mathematical error(s) are present	Some evidence is missing <u>or</u> minor mathematical error(s) are present	Sufficient evidence is presented <u>and</u> there are no mathematica l error(s)
3. The **warrants** connect the evidence to the claim. (Note that some quality mathematical arguments may not include a warrant.)	Warrants can take the form of definitions, theorems, logical inferences, and agreed upon facts. Warrants collectively chain the evidence together to show the claim is true or false. *Example:* I know these fractions are equivalent because the shaded area for each equivalent fraction is the same amount. *Non-example:* These fractions are equivalent because they are equal.	No warrant	Minimal support for evidence, <u>or</u> warrant unrelated to evidence is included <u>or</u> major conceptual error(s) are evident	Some evidence lacks a necessary warrant <u>or</u> minor conceptual error(s) are evident	Sufficient warrant <u>and</u> no conceptual error(s)
4. The **mechanics** help convey precise ideas that flow.	The language used must be at a sufficient level of precision to support the argument and with sufficient clarity. *Example:* ½, 2/4, 3/6, 4/8 are equivalent. Since the areas of the fractions all show the same amount those fractions must be equivalent. *Non-example:* They are the same.	The language has major imprecisions <u>or</u> does not flow, thus the ideas are unclear	The language has some imprecisions <u>or</u> thus the ideas are somewhat clear, thus the ideas are somewhat unclear but can be inferred	The language is precise <u>and</u> the ideas flow clearly	---

Key Connecting Sorting Packet to Argumentation Resource Packet

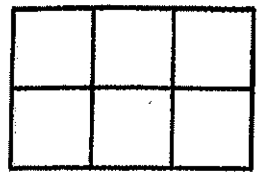
Student number (Sorting Packet)	Resource Packet Sample
1	C
2	A
3	B
4	
5	
6	
7	
8	
9	

Student number (Sorting Packet)	Resource Packet Sample (category)
2	A (high)
3	B (adequate)
1	C (low)
	D ( )
	E ( )
	F ( )
	G ( )
	H ( )
	I ( )

a. A small square is a square unit. What is the area of this rectangle? ~~Explain.~~

To get the area you multiply length × width. (2) × (3)

I got 6^2 units



2^2 units × 3^2 units = 6 sq. u

6 sq. units

b. What fraction of the area of each rectangle is shaded blue? Name the fraction in as many ways as you can. Explain your answers.

$A = \frac{1}{6} = \frac{2}{12} = \frac{4}{24} = \frac{8}{48} = \frac{16}{96} = \frac{32}{202}$

$B = \frac{1}{2}$ same as E, F, G

$C = \frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{8}{24} = \frac{16}{48} = \frac{32}{96}$

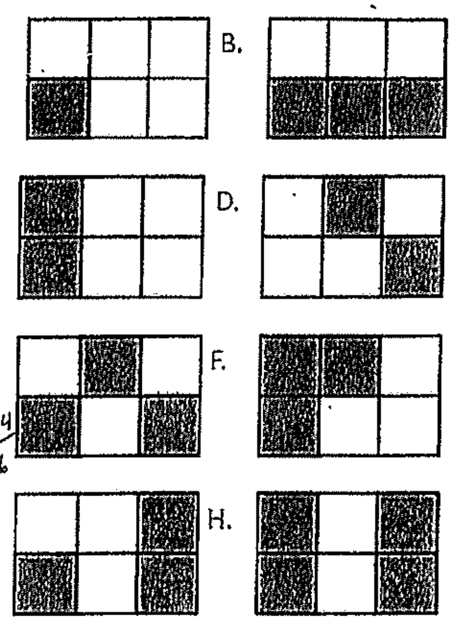
$D = \frac{1}{3}$ same as C

$E = \frac{1}{2}$ same as F and G

$F = \frac{1}{2} = \frac{3}{6} =$

$G = \frac{1}{2} = \frac{3}{6} = \frac{2}{4} = \frac{6}{12} = \frac{8}{16} = \frac{10}{20}$

$H = \frac{2}{3} = \frac{4}{6} = \frac{8}{12} = \frac{16}{24} = \frac{32}{48} = \frac{64}{96}$

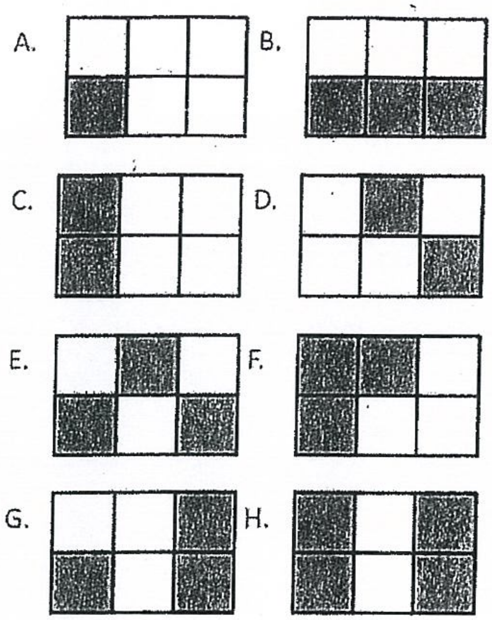


I got all the equivilent fractions because I multiplied all the fractions by $\frac{2}{2}$. To get my first solution by taking the one unit which was 6 boxes and counted all the colored boxes to get $\frac{1}{6}$ which means 1 out of 6 pieces. Then I multiplyed that by $\frac{2}{2}$.

6 square units

2^2 units x 3^2 units = 6^2 uni sq

b. What fraction of the area of each rectangle is shaded ~~blue~~? gray Name the fraction in as many ways as you can. Explain your answers.



A = 1/6 = 2/12 = 4/24 = 5/30 = 3/18 = 6/36

B = 3/6 = 1/2

C = 2/6 = 1/3

D = 2/6 = 1/3 = same as C

E = 3/6 = 1/2 = same as B

F = 3/6 = 1/2 = same as B

G = 3/6 = 1/2 = same as B

H = 4/6 = 2/3 = 8/12 = 12/18 = 16/24 =

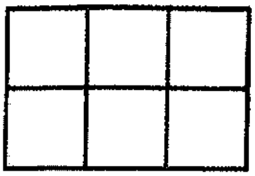
JJ

I multplied by a form of one to get each fraction. I storted by multipling by 2/2 then 3/3 then 4/4 then 5/5 and finally 6/6. For the first box, I counted the amout of squares in the retangle then I counted the shaded boxes. I got 1/6 for the first example.
~~I did my sequence of multipling fractions.~~

a. A small square is a square unit. What is the area of this rectangle? Explain.

Student 3

I think area of the rectangle is 6 square units. I know this because the rectangle



is split up into 6 small squares and it said that a small square is a square unit.

b. What fraction of the area of each rectangle is shaded blue? Name the fraction in as many ways as you can. Explain your answers.

A. $\frac{1}{6}, \frac{2}{12}, \frac{4}{24}, \frac{3}{18}$

B. $\frac{1}{2}, \frac{3}{6}, \frac{9}{18}, \frac{6}{12}$

C. $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}, \frac{6}{18}$

D. $\frac{2}{6}, \frac{1}{3}, \frac{6}{18}, \frac{4}{12}$

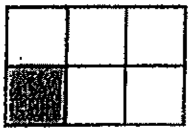
E. $\frac{1}{2}, \frac{3}{6}, \frac{50}{100}, \frac{9}{18}$

F. $\frac{3}{6}, \frac{1}{2}, \frac{50}{100}, \frac{9}{18}$

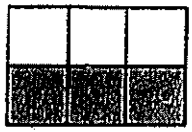
G. $\frac{1}{2}, \frac{3}{6}, \frac{500}{1000}, \frac{6}{12}$

H. $\frac{4}{6}, \frac{8}{12}, \frac{12}{18}, \frac{16}{24}$

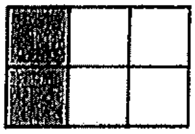
A.



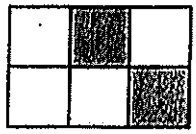
B.



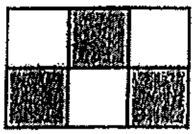
C.



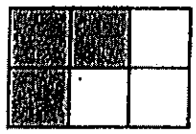
D.



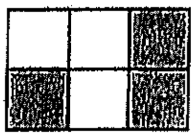
E.



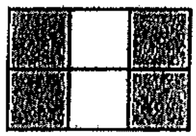
F.



G.



H.



I think to find the fraction of the shape, I looked at how many parts the rectangle was split into. That would be the denominator(6). Then I looked at how many parts was shaded, and that wou be the numerator(1). ~~To find the equivelent fraction I would double the numerator and denominator. (multiply by $\frac{2}{2}$)~~ or

One way is ~~Another way~~ to multiply by a form of one. Ex: $\frac{3}{6} \cdot \frac{3}{3} = \frac{9}{18}$

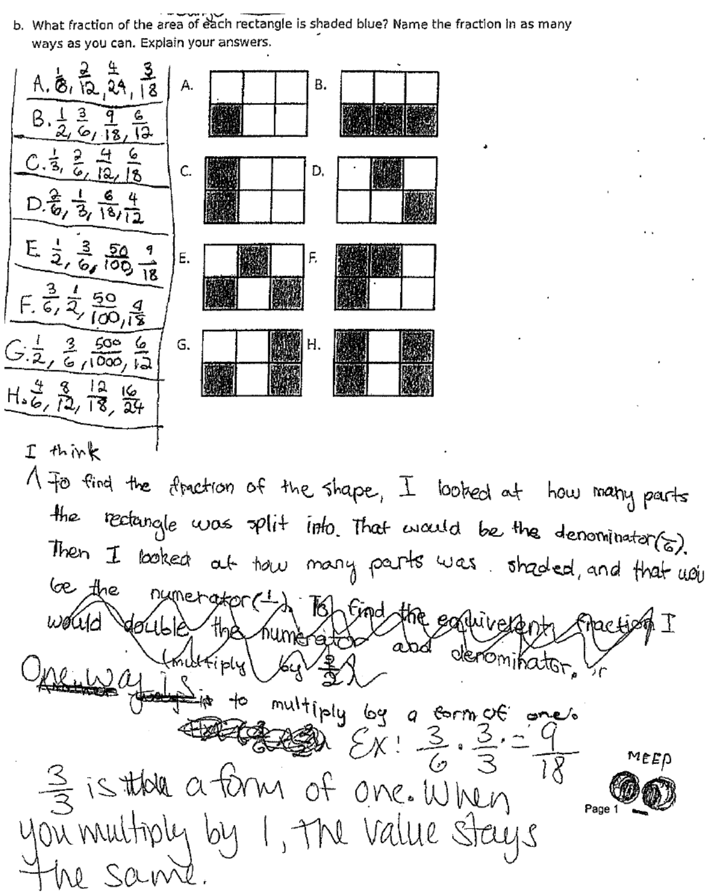
MEEP

$\frac{3}{3}$ is ~~tha~~ a form of one. When you multiply by 1, the value stays the same.

This will not be in the final packet. This is for our records here.

- Task title: Halves, Thirds and Sixths
- Grade level of task: 5
- Team members' names: Michael DiCicco and Brenda Moulton

Student A


b. What fraction of the area of each rectangle is shaded blue? Name the fraction in as many ways as you can. Explain your answers.

A. $\frac{1}{6}, \frac{2}{12}, \frac{4}{24}, \frac{3}{18}$

B. $\frac{1}{2}, \frac{3}{6}, \frac{9}{18}, \frac{6}{12}$

C. $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}, \frac{6}{18}$

D. $\frac{2}{6}, \frac{1}{3}, \frac{6}{18}, \frac{4}{12}$

E. $\frac{1}{2}, \frac{3}{6}, \frac{50}{100}, \frac{9}{18}$

F. $\frac{3}{6}, \frac{1}{2}, \frac{50}{100}, \frac{4}{18}$

G. $\frac{1}{2}, \frac{3}{6}, \frac{500}{1000}, \frac{6}{12}$

H. $\frac{4}{6}, \frac{8}{12}, \frac{12}{18}, \frac{16}{24}$

I think To find the fraction of the shape, I looked at how many parts the rectangle was split into. That would be the denominator($\frac{}{6}$). Then I looked at how many parts was shaded, and that would be the numerator($\frac{1}{}$). ~~To find the equivalent fraction I would double the numerator and denominator, or (multiply by $\frac{2}{2}$)~~

One way is to multiply by a form of one. Ex: $\frac{3}{6} \cdot \frac{3}{3} = \frac{9}{18}$

$\frac{3}{3}$ is a form of one. When you multiply by 1, the value stays the same.

MEEP

Page 1

Commentary

This student's argument was categorized as **High Quality.**

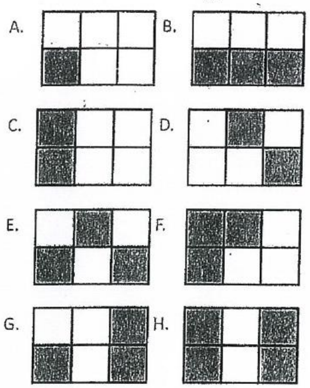
Student A's claim is that all of the fractions shown are equivalent to the corresponding fractions shown in the diagrams. Student A uses the multiplicative identity (multiplying by a form of 1) to show that 3/6 is equal to 9/18. The response generalizes why multiplying by a form of 1 results in an equivalent fraction.

Argumentation Components

Claim	Evidence
Implicit claim: all of the fractions shown in each part are equivalent	- 3/6 x 3/3 = 9/18 and - Given solutions
Warrants	**Language & Computation**
One way is to multiply by a form of 1. 3/3 is a form of 1. When you multiply by 1 the value stays the same.	The mathematical language used is precise and ideas flow clearly. Computations are correct.

Student B

…n of the area of each rectangle is shaded ~~blue~~ gray? Name the fraction in as many … can. Explain your answers.



A = 1/6 = 2/12 = 4/24 = 5/30 = 3/18 = 6/36

B = 3/6 = 1/2

C = 2/6 = 1/3

D = 2/6 = 1/3 = same as C

E = 3/6 = 1/2 = same as B

F = 3/6 = 1/2 = same as B

G = 3/6 = 1/2 = same as B

H = 4/6 = 2/3 = 8/12 = 12/18 = 16/24 =

I multipled by a form of one to get each fraction. I started by multipling by 2/2 then 3/3 then 4/4 then 5/5 and finally 6/6. For the first box, I counted the amount of squares in the retangle then I counted the shaded boxes, I got 1/6 for the first example. ~~I did my sequence of multipling fractions.~~

Commentary

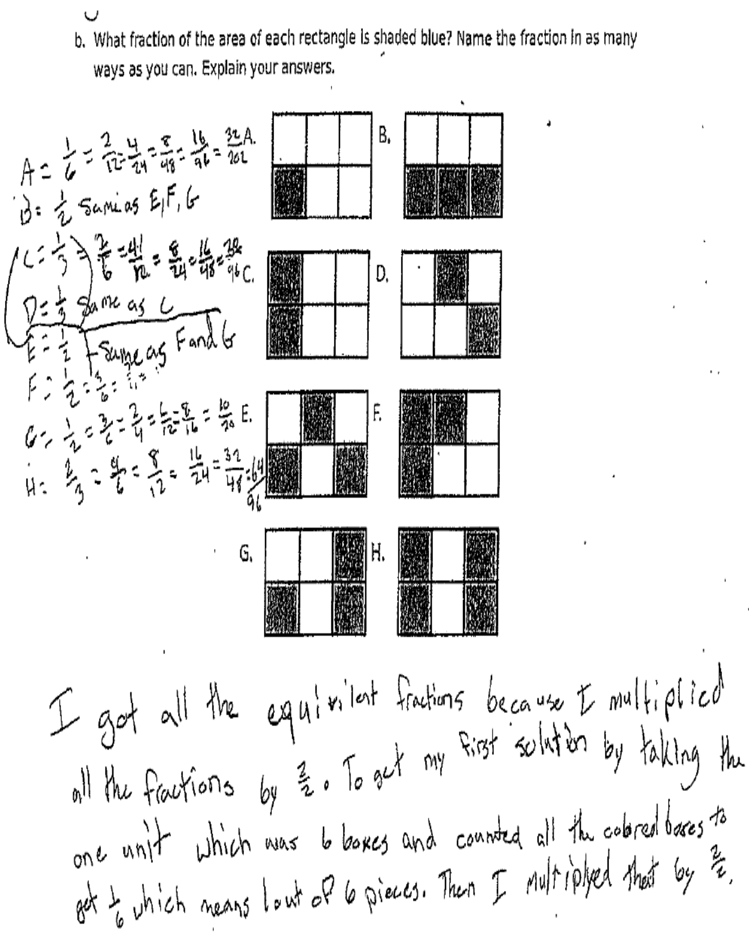
This student's argument was categorized as **Adequate Quality.**

Student B's claim is that all of the fractions shown are equivalent to the corresponding fractions shown in the diagrams. Student B states that by multiplying by forms of 1, equivalent fractions are formed. However, the response does not explain why multiplying by a form of 1 results in an equivalent fraction. The argument could be strengthened by supporting the statement "multiplication by a form of 1" explaining that this multiplication does not change the value of the fractions (multiplicative identity).

Argumentation Components

Claim	**Evidence**
Implicit claim: all of the fractions shown in each part are equivalent	Given solutions
Warrants	**Language & Computation**
(See written explanation at bottom of student's work)	The mathematical language used is precise. Computations are correct.

Student C



Commentary

This student's argument was categorized as **Low Quality.**

Student C's claim is that all of the fractions shown are equivalent to the corresponding fractions shown in the diagrams. Student C only states that multiplying by 2/2 generates equivalent fractions. However, no support is given for why this approach is viable.

The argument would be strengthened by explaining that 2/2 is a form of 1 and therefore it can be used to find equivalent fractions. The argument should also contain an explanation for why multiplying by a form of 1 results in an equivalent fraction.

Argumentation Components

Claim	**Evidence**
Implicit claim: all of the fractions shown in each part are equivalent	Given solutions
Warrants	**Language & Computation**
(See written text at bottom of student's work)	The mathematical language used is precise. Computations are correct.

Rubric

Category	Description with Examples/Non-Examples	0	1	2	3
1. The **claim** presents the position being taken.	The claim is what is to be shown true or not true. *Example:* The fractions shown are equivalent to the corresponding fractions shown in the diagrams. *Non-example:* no equivalent fractions are given	No claim	Claim is included but not clear	Claim is clearly articulated	---
2. **Evidence** supports the claim.	Evidence can take the form of equations, tables, charts, diagrams, graphs, words, symbols, etc. It is one's "work" which provides the information to show something is true/false. *Example:* 3/6 x 3/3 = 9/18 *Non-example:* 3/6 = 9/18	No evidence	Minimal evidence is included, <u>or</u> evidence is unrelated to the claim, <u>or</u> major mathematical error(s) are present	Some evidence is missing <u>or</u> minor mathematical error(s) are present	Sufficient evidence is presented <u>and</u> there are no mathematica l error(s)
3. The **warrants** connect the evidence to the claim. (Note that some quality mathematical arguments may not include a warrant.)	Warrants can take the form of definitions, theorems, logical inferences, and agreed upon facts. Warrants collectively chain the evidence together to show the claim is true or false. *Example:* One way is to multiply by a form of 1. 3/3 is a form of 1. When you multiply by one the value stays the same. *Non-example:* Multiply by 3/3 to get an equivalent fraction.	No warrant	Minimal support for evidence, <u>or</u> warrant unrelated to evidence is included <u>or</u> major conceptual error(s) are evident	Some evidence lacks a necessary warrant <u>or</u> minor conceptual error(s) are evident	Sufficient warrant <u>and</u> no conceptual error(s)
4. The **mechanics** help convey precise ideas that flow.	The language used must be at a sufficient level of precision to support the argument and with sufficient clarity. *Example:* To find the fraction of the shape, I looked at how many parts the rectangle was split into. That is the denominator. Then I looked at how many parts were shaded in. That is the numerator. *Non-example:* To find the fraction I looked at the picture and how much was shaded. (Note the lack of precision with language.)	The language has major imprecisions <u>or</u> does not flow, thus the ideas are unclear	The language has some imprecisions <u>or</u> thus the ideas are somewhat clear, thus the ideas are somewhat unclear but can be inferred	The language is precise <u>and</u> the ideas flow clearly	---

Key Connecting Sorting Packet to Argumentation Resource Packet

Student number (Soring Packet)	Resource Packet Sample
1	C
2	B
3	A
4	
5	
6	
7	
8	
9	

Student number (Soring Packet)	Resource Packet Sample (category)
3	A (high)
2	B (adequate)
1	C (low)
	D ( )
	E ( )
	F ( )
	G ( )
	H ( )
	I ( )

Name: [redacted]

EQUIVALENCY ARGUMENT

Find a fraction equivalent to 3/8. Use diagrams, equations, and mathematical principles to prove that the fractions are equivalent.

Make sure your argument includes a claim, evidence, warrants, reasoning and conclusion.

Claim: The answer is 6/16

Evidence:

3/8 = 6/16 ~~Because~~

and Reasoning

Warrent: 6/16 is the right answer because if you times ~~it~~ the fraction by 1 (which is $\frac{2}{2}$) you get 6/16 it is the same because the numerator and the denomoator ~~the~~ is times by 1 ($\frac{2}{2}$) so it will be the same value. Any thing ~~th~~ times 1 is the same value

Conclusion: 6/16 is equal to 3/8 because $\frac{2}{2}$ is equal to 1 and anything times 1 is the same value so...

$$\frac{3}{8} \times \frac{2}{2} = \frac{6}{16}$$

so this why $\frac{3}{8}$ is equalent to 6/16

EQUIVALENCY ARGUMENT

Find a fraction equivalent to 3/8. Use diagrams, equations, and mathematical principles to prove that the fractions are equivalent.

Make sure your argument includes a claim, evidence, warrants, reasoning and conclusion.

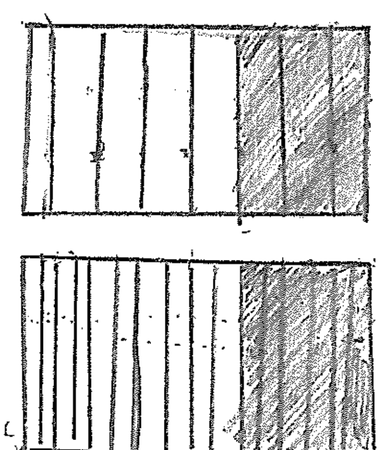
I believe that there is a fraction equivelent to 3/8.
One possibe equivelent fraction is 6/16. This is proven by the equation and diagram below.

Equation

$$\frac{3}{8} \cdot \frac{2}{2} = \frac{6}{16}$$

This works because 2/2 is equal to 1 or the giant 1. Also you are multiplying the numerator and denominator by the same thing.

Diagram



3/8

6/16

So as you can see, 3/8 can easily be charge to an equivlent fraction.

Nam[redacted]

EQUIVALENCY ARGUMENT

Find a fraction equivalent to 3/8. Use diagrams, equations, and mathematical principles to prove that the fractions are equivalent.

Make sure your argument includes a ~~claim~~, ~~evidence, warrants,~~ ~~reasoning and conclusi~~on.

Claim - $\frac{6}{16}$ it is just dubled

evidence - $\frac{3 \times 2 = 6}{8 \times 2 = 16}$ not a hole they are both not holes

warants - $\frac{3 \times 2 = 6}{8 \times 2 = 16}$

conclusion / reasoning - The numbers are Just dobled and are not hole.

This will not be in the final packet.
This is for our records here.

- Task title: Equivalency Argument
- Grade level of task: 6
- Team members' names: Jeff Burnham and Jocelyn Dunnack

Student A

EQUIVALENCY ARGUMENT

Find a fraction equivalent to 3/8. Use diagrams, equations, and mathematical principles to prove that the fractions are equivalent.

Make sure your argument includes a claim, evidence, warrants, reasoning and conclusion.

Claim: The answer is $6/16$

Evidence:

$3/8 = 6/16$ ~~Because~~

and Reasoning

Warrent: $6/16$ is the right answer because if you times the fraction ~~it~~ by ~~2~~ 1 (which is $\frac{2}{2}$) you get $6/16$ it is the same because the numerator and the denomoator ~~the~~ is times by ~~2~~ 1 $(\frac{2}{2})$ so it will be the same value. Any thing ~~is~~ times 1 is the same value

Conclusion: $6/16$ is equal to $3/8$ because $\frac{2}{2}$ is equal to 1 and anything times 1 is the same value so...

$$\frac{3}{8}\times\frac{2}{2}=\frac{6}{16}$$

So this why $\frac{3}{8}$ is equalent to $6/16$

Commentary

This student's argument was categorized as **High Quality**.

Because this task was familiar for 6th graders, most students, including this one, were able to find a correct claim and provide evidence.

This student states that 2/2 is equal to 1 and states that multiplying by one creates an equivalent value. Even though this example is brief, it included a clear claim, evidence, and warrant.

In general, High Quality arguments explicitly stated the warrant that multiplying by one doesn't change the value of the fraction. Students work with this concept for several years before 6th grade, and this warrant reflects deep understanding of equivalent fractions and strong support for creating equivalent fractions.

Argumentation Components

Claim	Evidence
"The answer is 6/16." Note: A clearer way to say this might be "6/16=3/8", but the claim is clear.	The student's evidence is the equation "3/8 x 2/2 = 6/16". Note: Due to the brevity of the assignment, this is sufficient to support the claim.
Warrants	**Language & Computation**
This student states that 2/2 is equal to 1 and states that multiplying by one creates an equivalent value. Note: While the principle is not named, this student clearly understands Multiplicative Identity.	There is an instance of incorrect use of mathematical language: "times" is used for multiply. The student's revisions show the student started to say you multiply by 2, but then realized it must be said that 2/2 is 1. The warrant is clear and concise.

Student B

EQUIVALENCY ARGUMENT

Find a fraction equivalent to 3/8. Use diagrams, equations, and mathematical principles to prove that the fractions are equivalent.

Make sure your argument includes a claim, evidence, warrants, reasoning and conclusion.

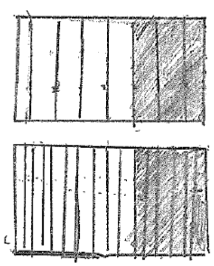
I believe that there is a fraction equivelent to 3/8.
One possibe equivelent fraction is 6/16. This is proven by the equation and diagram below.

Equation

$$\frac{3}{8} \cdot \frac{2}{2} = \frac{6}{16}$$

This works because 2/2 is equal to 1 or the giant 1, Also you are multiplying the numerator and denominator by the same thing.

Diagram



3/8

6/16

So as you can see, 3/8 can easily be Change to an equivilent fraction.

Commentary

This student's argument was categorized as **Adequate Quality.**

Because this task was familiar for 6th graders, most students, including this one, were able to find a correct claim and provide evidence.

This student states that 2/2 is equal to 1 but doesn't explain the importance of multiplying by one to find an equivalent fraction. This student also included an accurate diagram as further evidence, but didn't explicitly connect the diagram to the claim with a warrant (the shaded areas are equal).

In general, Adequate Quality arguments tended to have implied or incomplete warrants.

Argumentation Components

Claim	Evidence
"One possible equivalent fraction is 6/16."	This student provides an equation and a diagram to support the claim. The diagram is accurate and clear. The equation is correct.
Warrants	**Language & Computation**
This student has incomplete warrants. This student states that 2/2 is equal to 1 but doesn't explain the importance of multiplying by one to find an equivalent fraction.	This is well written, but the chain of reasoning is missing the warrants. The reader must imply the warrant from the diagrams.

Student C

EQUIVALENCY ARGUMENT

Find a fraction equivalent to 3/8. Use diagrams, equations, and mathematical principles to prove that the fractions are equivalent.

Make sure your argument includes a claim, ~~evidence, warrants, reasoning and conclusion~~.

Claim – $\frac{6}{16}$ it is just dubled

evidence – $\frac{3 \times 2 = 6}{8 \times 2 = 16}$ not a hole they are both not holes

warants – $\frac{3 \times 2 = 6}{8 \times 2 = 16}$

conclusion / reasoning – The numbers are Just dobled and are not hole.

Commentary

This student's argument was categorized as **Low Quality**.

Because this task was familiar for 6th graders, most students, including this one, were able to find a correct claim and provide evidence.

This student incorrectly stated that the fraction was doubled. The student doesn't explicitly demonstrate understanding of how multiplying by a form of 1 generates an equivalent fraction, even though the evidence implies understanding, or at least the ability to use the algorithm.

In general, Low Quality arguments tended to have faulty warrants.

Argumentation Components

Claim	Evidence
A claim of "6/16" is correct, but could be stated more completely.	Evidence shows use of multiplicative identity, although it is imprecisely expressed under "Evidence" and more accurately expressed under "Warrants".
Warrants	**Language & Computation**
The warrant is faulty. The student states that "6/16 it is just doubled." There is no mention of multiplying by 1 to find equivalent fractions. The student tries to use another warrant, that both fractions are still less than 1 whole, but it is not appropriate here.	There is an instance of incorrect spelling: "hole" is used for whole. The calculations are correct and the student restates the warrants, which is a good strategy for writing a clear argument.

Key Connecting Sorting Packet to Argumentation Resource Packet

Student number (Sorting Packet)	Resource Packet Sample
1	A
2	B
3	C
4	
5	
6	
7	
8	
9	

Student number (Sorting Packet)	Resource Packet Sample (category)
1	A (high)
2	B (adequate)
3	C (low)
	D ( )
	E ( )
	F ( )
	G ( )
	H ( )
	I ( )