# *What is Mathematical Argumentation?*




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ATOMIC
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# WARM UP!



## What is a mathematical argument?

*As an example, create an argument for the following. This may help you think through what a mathematical argument is…*

*When you add any two consecutive numbers, the answer is always odd.*

Think

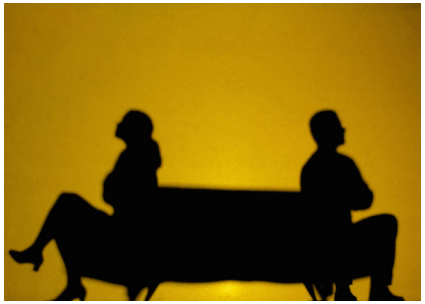
1) Is this statement (claim) true?
2) What's your argument to show that it is or is not true?

*Introduce yourself to your neighbor*
*Share some ideas from what you wrote*


justifying
logical chain
showing to be true
reasoning
convincing
evidence
communicating

# One way to think about arguments





# Another way to think about arguments…

avoid, avoid, avoid



# THIS IS NOT WHAT WE MEAN!

# A mathematical argument

It is…

- A sequence of statements and reasons given with the aim of demonstrating that a claim is true or false

It is not…

- Solely an explanation of what you did (steps)
- A recounting of your problem solving process
- Explaining why you personally think it's true for reasons that are not necessarily mathematical (e.g., popular consensus; external authority, etc. *It's true because John said it, and he's always right.*)

# Standards of Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Let's consider

# THE SUM OF TWO CONSECUTIVE NUMBERS IS ODD

*When you add any two consecutive numbers, the answer is always odd.*

# Discuss: What is each student's argument?

# Which argument(s) show the claim is true?

*How are they similar?*
*How are they different?*

# *When you add any two consecutive numbers, the answer is always odd.*

*Micah's Response*

5 and 6 are consecutive numbers, and 5 + 6 = 11 and 11 is an odd number.

12 and 13 are consecutive numbers, and 12 + 13 = 25 and 25 is an odd number.

1240 and 1241 are consecutive numbers, and 1240 +1241 = 2481 and 2481 is an odd number.

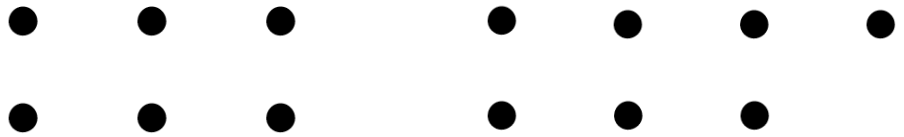
That's how I know that no matter what two consecutive numbers you add, the answer will always be an odd number.

# *When you add any two consecutive numbers, the answer is always odd.*

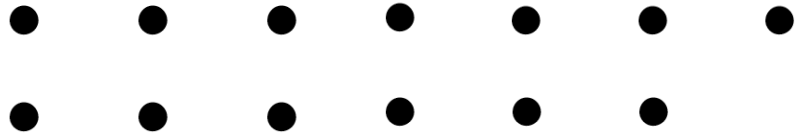
*Roland's Response*

The answer is always odd.

A number + The next number =



An odd number



There's always one left over when you put them together, so it's odd.

# *When you add any two consecutive numbers, the answer is always odd.*

## *Angel's Response*

Consecutive numbers go even, odd, even, odd, and so on. So if you take any two consecutive numbers, you will always get one even and one odd number.

And we know that when you add any even number with any odd number the answer is always odd.

That's how I know that no matter what two consecutive numbers you add, the answer will always be an odd number.

# *When you add any two consecutive numbers, the answer is always odd.*

*Kira's Response*

Consecutive numbers are $n$ and $n+1$.

Add the two numbers:
$n + (n+1) = 2n + 1$

You get $2n + 1$ which is always an odd number, because an odd number leaves a remainder of 1 when divided by 2. (2 goes into $2n + 1$ $n$ times, with a remainder of 1)

# Comments on the approaches

- Example-based (Micah)
- Pictorial (Roland)
- Narrative (Angel)
- Symbolic/algebraic (Kira)

# Summary Take-aways

- Mathematical argument: logical chain to show it is true or false
- There are many ways to construct an argument!
- Requires using understandings of consecutive numbers and odd/even numbers, or more generally, key math ideas.
- Empirical – good beginning; not enough. Need to address question: *Why MUST it be so?*
  - *Counter example*

# NUMBER TALK -- THINK!



16 x 25

What's the difference between giving an *explanation of one's steps* and giving a *mathematical argument*?

*When you have a strategy for finding the answer, show me one finger in front of you.*
*Keep thinking.*
*If you get a second strategy, show 2 fingers.*

# Explanation of Steps vs Argument

Student Work

16 = 10 + 6
10 x 25 = 250
6 x 25 = 150
250 + 150 = 400

Explanation of Steps

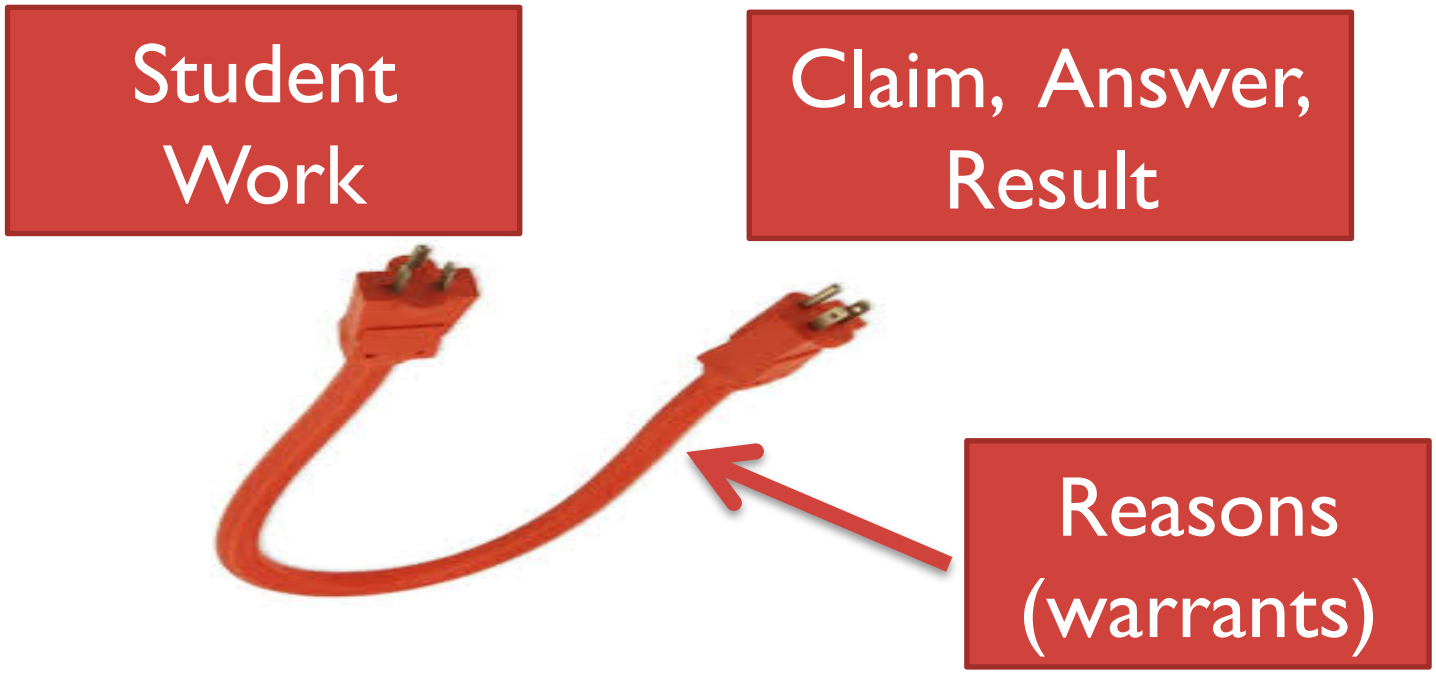
I took 16 and split it into 10 and 6. I multiplied 10 by 25, and I multiplied 6 by 25. And then I added those 2 numbers together.

Argument

I took 16 and split it into 10 and 6 *because I need to find 16 groups of 25, and so I can find 10 groups of 25 and then add to it 6 more groups of 25*. So I multiplied 10 by 25 and 6 by 25. I added those two numbers together *to give me 16 25s total, which is what I need.*

# Summary Take-aways

- Arguments are *beyond* sharing steps and showing your work
- Arguments include the ***reasons*** (warrants, connectors) that link student's work to a claim (the claim, answer, what is true)



# ARGUMENTATION AS SOMETHING STUDENTS *DO* IN THE CLASSROOM

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# Constructing viable arguments

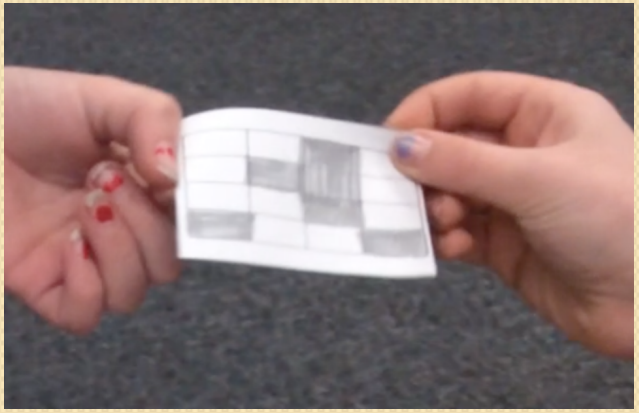
- *use* stated assumptions, definitions, and previously established results in constructing arguments.
- *make* conjectures, including *reason inductively* about data
- *build* a logical progression of statements to explore the truth of their conjectures
- *analyze* situations by breaking them into cases
- *recognize* and *use* counterexamples
- *justify* their conclusions
- *communicate* them to others
- *respond* to the arguments of others
- *distinguish* correct logic or reasoning from that which is flawed

Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.

**Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.**

# Welcome to Argumentation in Grade 3

- Students have worked on a sorting task with the focus on "Is it a Half?"
- These video clips are from one lesson in the middle of the school year, about two weeks into a unit on fractions
- The purpose of the lesson was to understand half in many different ways
- The lesson task is available at: http://bridges.education.uconn.edu/repository/



# ARGUMENTATION IN ACTION

## *Is It A Half?*

**Sarah Brown's 3rd-Grade Class**

## IS IT A HALF

# Lesson Plan

"At the beginning of the class, we brainstormed a list of things that were "a half." Students said things like half and apple, half moon, half full. I paired them up for this activity not by ability but to be sure there would be some kind of give-and-take with pair because I wanted the argumentation to come out and not one person doing everything. The students communicated orally with their partners to complete this activity. Students had about 20-25 minutes to work on this. Then we came together for a whole group discussion."

# Questions for Clip 1

Guiding Questions

a. What did Sarah do to support argumentation?
b. What norms and routines were in place to support the students' participation in argumentation?
c. What was the student's argument?

# Clip 2

*The class is asked if they agree or disagree.*
*The student looks to support from her partner.*

# Questions for Clip 2

Guiding Questions

a. What norms and routines were in place to support the students' participation in argumentation?
b. What is the role of the teacher?
c. What do you think students learned as a result of this exchange, or would learn as a result of exchanges like this?

# Establishing Norms and Routines to Support Argumentation

# Sharing Tools/Supports

- Types of tasks that support argumentation
- Modifications of tasks
  - Show your work vs. Defend
- Refer to task repository
  - Argumentation doesn't need to be something new
- Graphic organizers

It's not 'one more thing.' It's making what you do better.

# Turn and Talk

**3** things you discovered about argumentation.

**2** things you might do in your classroom or professional work.

**1** question you still have.

# ? Questions ?



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# THANK YOU!

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