Comparing Fractions

STUDENT WORK SAMPLE ARGUMENTATION RESOURCE PACKET

This packet was produced as part of the Bridging Math Practices Math-Science Partnership Grant (2014 -2015).

The purpose of the packet is to help a) reveal what students can do with respect to generating an argument in response to mathematical questions, including the variety of their arguments; b) highlight features that should be considered when reviewing students' arguments, and c) identify what counts as a *quality* argument in light of the review criteria.

What is a mathematical argument?

A mathematical argument is
a sequence of statements and reasons given with the aim of demonstrating that a claim is true or false.

This links to the Connecticut Core Standards of Mathematical Practice #3, *construct viable arguments and critique the reasoning of others,* as well as other standards.

This resource packet is a product of work by participants in the UConn Bridging Math Practices Math-Science Partnership Grant, which included faculty and graduate students from the University of Connecticut's Neag School of Education and Department of Mathematics, and teachers and coaches from the Manchester Public Schools, Mansfield Public Schools, and Hartford Public Schools. This resource packet reflects significant contributions from Monica Braham, Pari Ghetia, Laura Kowaleski, Colleen Litwin, Michelle McKnight,Tracy Pietkevich. Many thanks for all their insights and contributions! For more information about the grant, or for additional argumentation-related materials and resource, please see the project website: http://bridges.uconn.education.edu
The Mathematics and Science Partnership (MSP) grant is a federal program funded under Title II, Part B, of the *Elementary and Secondary Education Act* and administered by the U.S. Department of Education (ED).

What is a high quality mathematical argument?

A high quality mathematical argument is an argument that shows that a claim must be true. It leaves little room to question. The chain of logic leads the read to conclude that the author's claim is true.

What are the characteristics of a high quality argument? A high quality argument can be described by the following components and criteria:

Criteria	**Description**
1. A clearly stated **claim**	The claim is what is to be shown true or not true.
2. The necessary **evidence** to support the claim	Evidence can take the form of equations, tables, charts, diagrams, graphs, words, symbols, etc. It is one's "work" which provides the information to show something is true/false.
3. The necessary **warrants** to connect the evidence to the claim	Warrants can take the form of definitions, theorems, logical inferences, agreed upon facts. Warrants explain how the evidence is relevant for the claim, and collectively they chain the evidence together to show the claim is true or false.
4. **Language use and computations** are at a sufficient level of precision and accuracy	The language used and computations must be at a sufficient level of precision or accuracy to support the argument. Language use needs to be precise enough to communicate the ideas with sufficient clarity.

These criteria are helpful for discussions. It is important not to lose sight of the "big picture" however, and that is whether the argument offered shows that the claim is (or is not) true. This is the goal and purpose of a mathematical argument. You will see in many of these packets that students can approach an argumentation prompt from many different perspectives. It matters less *which* mathematical tools they use, and matters more whether their chain of reasoning compels the result.

In this packet you will find

1. A blank copy of the task: (name of task) and a description of the implementation context or and/or other considerations about the work samples you will be analyzing.
2. A protocol that can help you and your colleagues discuss student work related to this task.
3. Selected work samples from 3rd grade students in classes of teacher participants in the UConn Bridging Math Practices project.
4. The student work samples ordered by whether they seem to be *high, adequate, or low quality* responses with respect to the above criteria; along with commentaries that support the classification. Among the samples are some that present a well-structured argument, but have important mathematical flaws, which prevent them from being classified as the highest quality.

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much "weight" people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional *discussions* and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

THE TASK

Comparing Fractions

Javier claims that 1/2 is less than 3/8.

Do you agree or disagree?

CONTEXT

This problem is a teacher-generated task. Students worked in groups to write a mathematical argument. Each sample on the following pages represent one group's collaborative efforts at creating an argument. The commentaries assume the statements in each sample represent one argument.

Protocol Guided Sorting Activity: (33–40 mins)
Bridging Math Practices Math-Science Partnership Grant

This protocol was created for the purpose of reviewing student work. It is modified from two of the previously presented protocols in the Manchester School District. The original protocols apply to when teachers bring their own students' work. This has been modified to review prepared packets of student work.

- Maryland Protocol: Examining Student Work to Inform Instruction – Maryland State Department of Education http://mdk12.org/instruction/examining/protocol.html
- Collaborative Analysis Protocol - San Diego County Board of Education http://plc.sdcoe.net/Resources/Data%20Driven%20Decisions/LASWProtocol_Dec2011Rev.pdf

This is sometimes referred to as a ***Tuning Protocol,*** as the purpose is to help a group align their visions and expectations. Here, the alignment is with respect to the question: what is a high quality argument (on this task, for this grade level)? A main goal of this protocol is to support colleagues in building a consensus around what counts as a high quality argument.

0. Assign Roles

The Handler – places work samples in agreed-upon pile
Facilitator – ensures space is made for all to contribute; supports finding consensus
Time Keeper – keeps time and ensures group doesn't exceed section time limits. Can prompt movement to next section even if full time is not used.
All– share ideas and keep notes on own set of work samples

A: Setting the context for discussion (5 mins)

Team members read and do the problem. Team members discuss: What was the "big idea" of the task/assessment? What result or claim needed justification?

B: Quick sort: Reviewing student work (15 mins)

Do a *Quick Sort* of students' work by the degree of proficiency (high, adequate, low) demonstrated with providing an argument of the relevant claim(s). The Handler places a copy of the student work into the appropriate pile as agreed upon by the group. You may initially need a "Not Sure" pile. After sorting, revisit papers in the "Not Sure" pile and match each with the typical papers in one of the other piles. Record work sample numbers in the appropriate column of the chart (next page).

The facilitator may also decide to begin the Quick Sort with some silent review of student work samples before starting discussion.

Sorting Chart

HIGH Quality (high quality mathematical argument)	**ADEQUATE Quality** (adequate mathematical argument)	**LOW** (low quality mathematical argument)

C: Strengths and areas for growth? (5 mins)

Group member summarize key ideas from their Sorting Discussion regarding the strengths and areas for growth for individual samples, each group[1] (High Quality, Adequate, Low) of samples, or the overall set with respect to the argumentation?

HIGH Quality (high quality mathematical argument)	**ADEQUATE Quality** (adequate mathematical argument)	**LOW** (low quality mathematical argument)
Strengths overall for the class		

[1] This question is phrased in terms of "subgroups." You may or may not be able to characterize the group as a whole. As needed, describe individual or pairs of student work.

D: Reading ARP Commentaries (optional: 5-7 mins)

As deemed useful, group members read the commentaries in the Argumentation Resource Packet to gain new perspectives on selected student work samples, their strengths and areas for growth, and what counts as a high quality argument.

E: Reflection (5 mins) *Each person shares*

The facilitator guides the group to take turns in sharing a reflection. Group may decide to reflect on the same question, or each share a take away.

a. What did you learn about argumentation and how students engage argumentation from looking at the work of these students? You might also consider aspects of task design.

b. Did you have any *ah hah* moments?

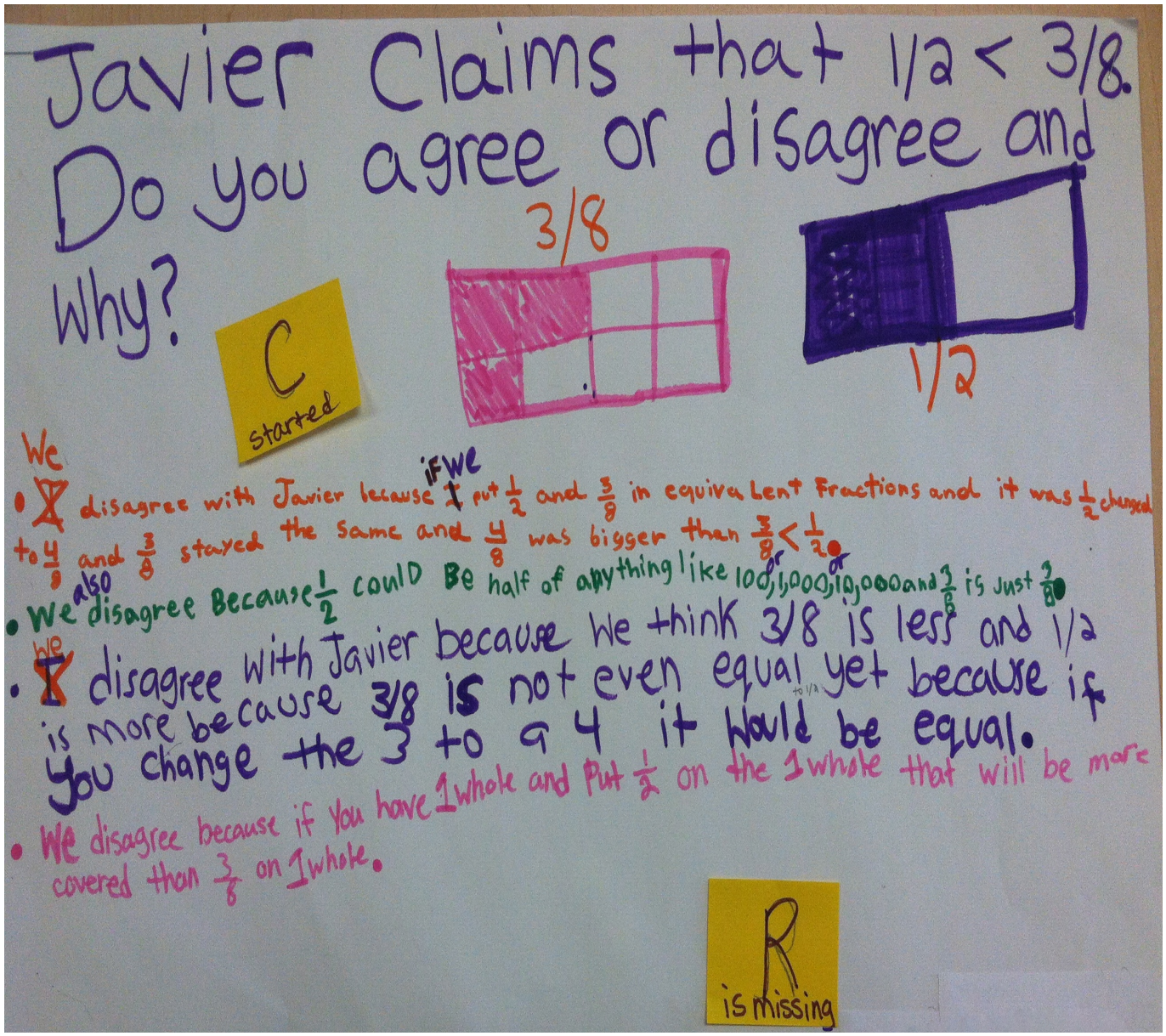
c. What questions remain for you? What would you like to lean more about?

d. What will you take away from this discussion back to your classroom? What ideas might impact your planning or teaching?

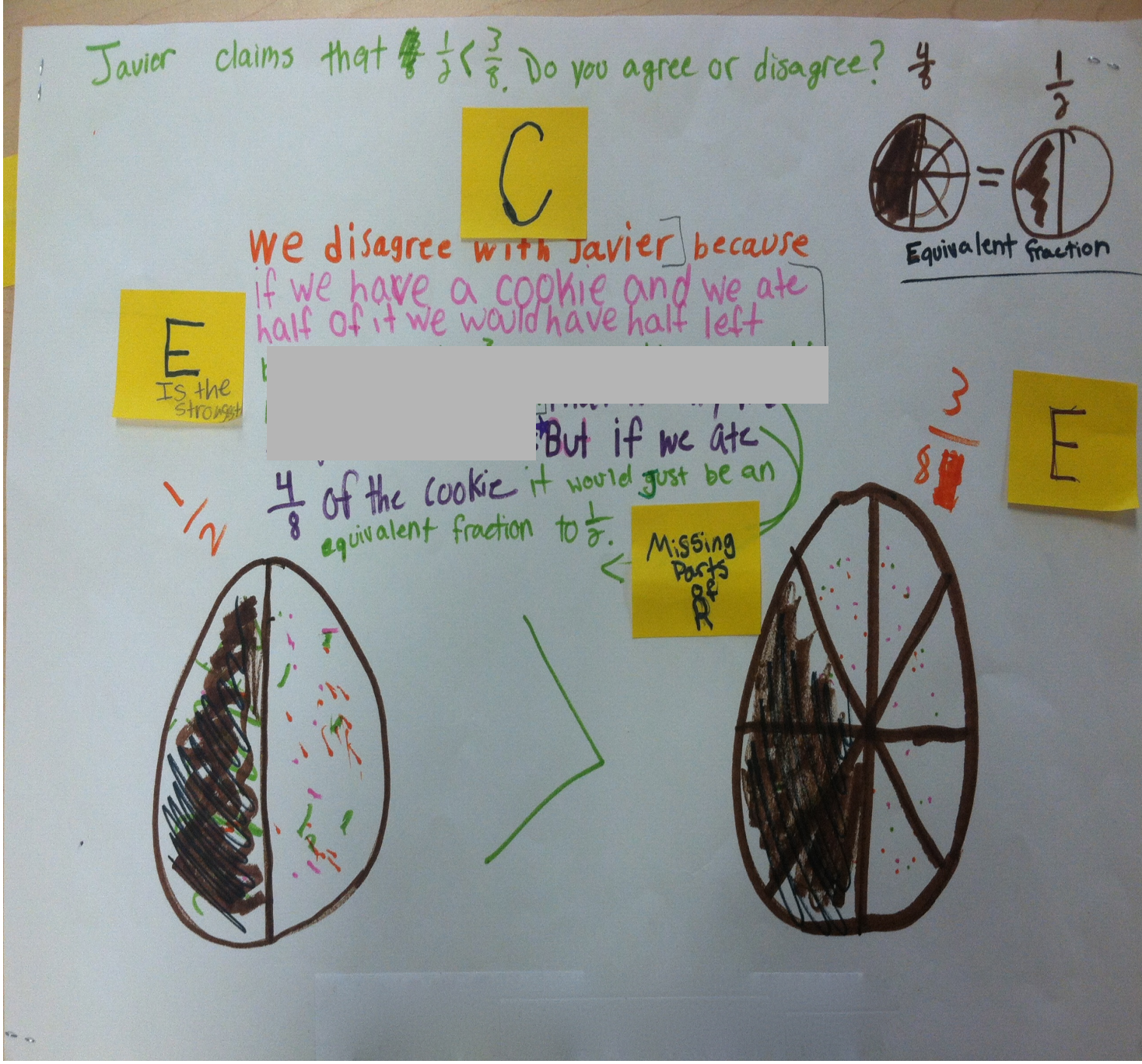
F: Reflection on Protocol Implementation (3 mins)

Facilitator guides a reflection on how the protocol process worked. Group members contribute ideas. Members make suggestions for modifications to future protocol as needed.

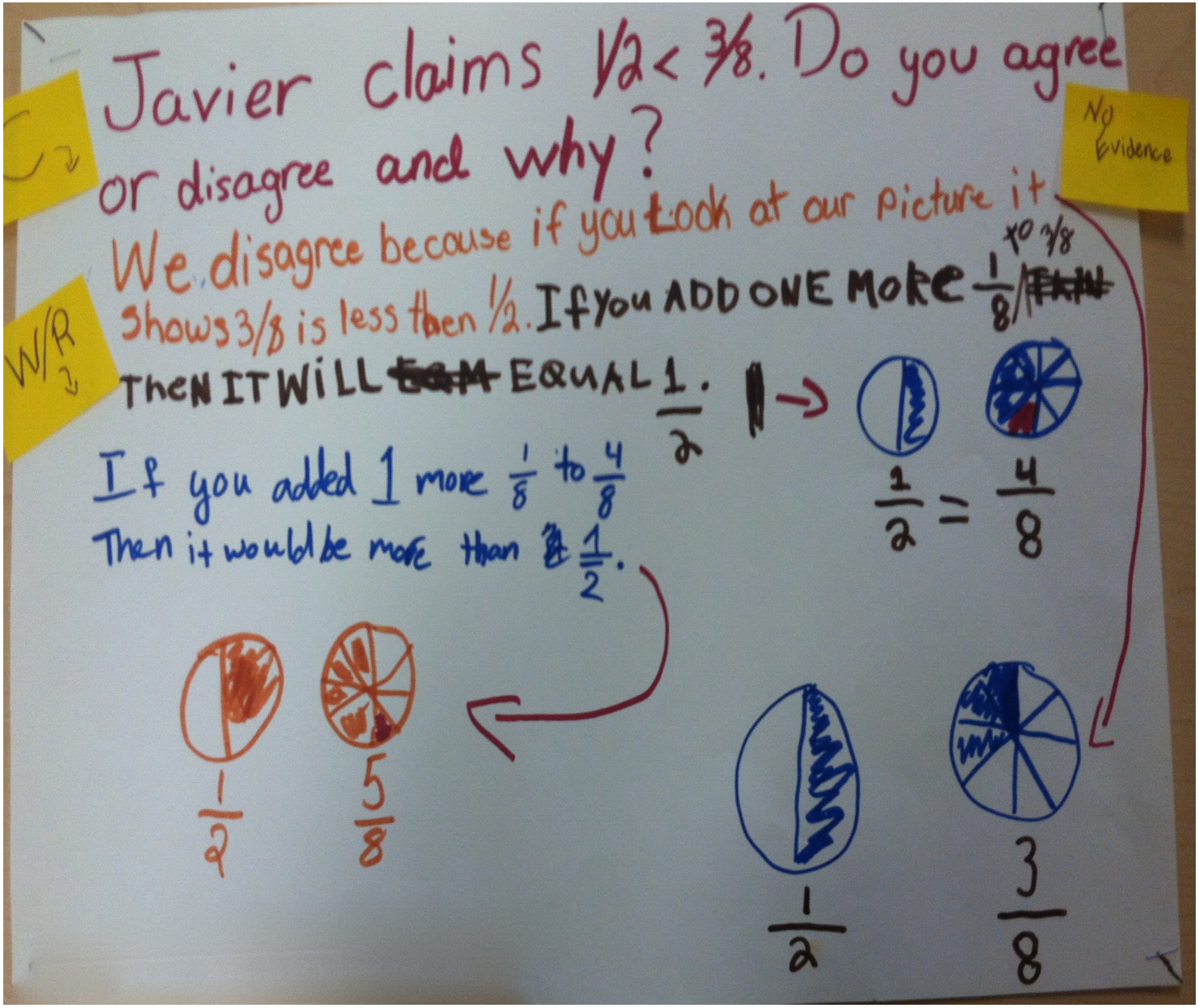
Student 1


Javier Claims that 1/2 < 3/8. Do you agree or disagree and Why?
3/8
1/2
C started
We disagree with Javier because if we put 1/2 and 3/8 in equivalent fractions and it was 1/2 changed to 4/8 and 3/8 stayed the same and 4/8 was bigger than 3/8 < 1/2.
We also disagree because 1/2 could be half of anything like 100, or 1,000, or 10,000 and 3/8 is just 3/8.
We disagree with Javier because we think 3/8 is less and 1/2 is more because 3/8 is not even equal to 1/2 yet because if you change the 3 to a 4 it would be equal.
We disagree because if you have 1 whole and put 1/2 on the 1 whole that will be more covered than 3/8 on 1 whole.
R is missing

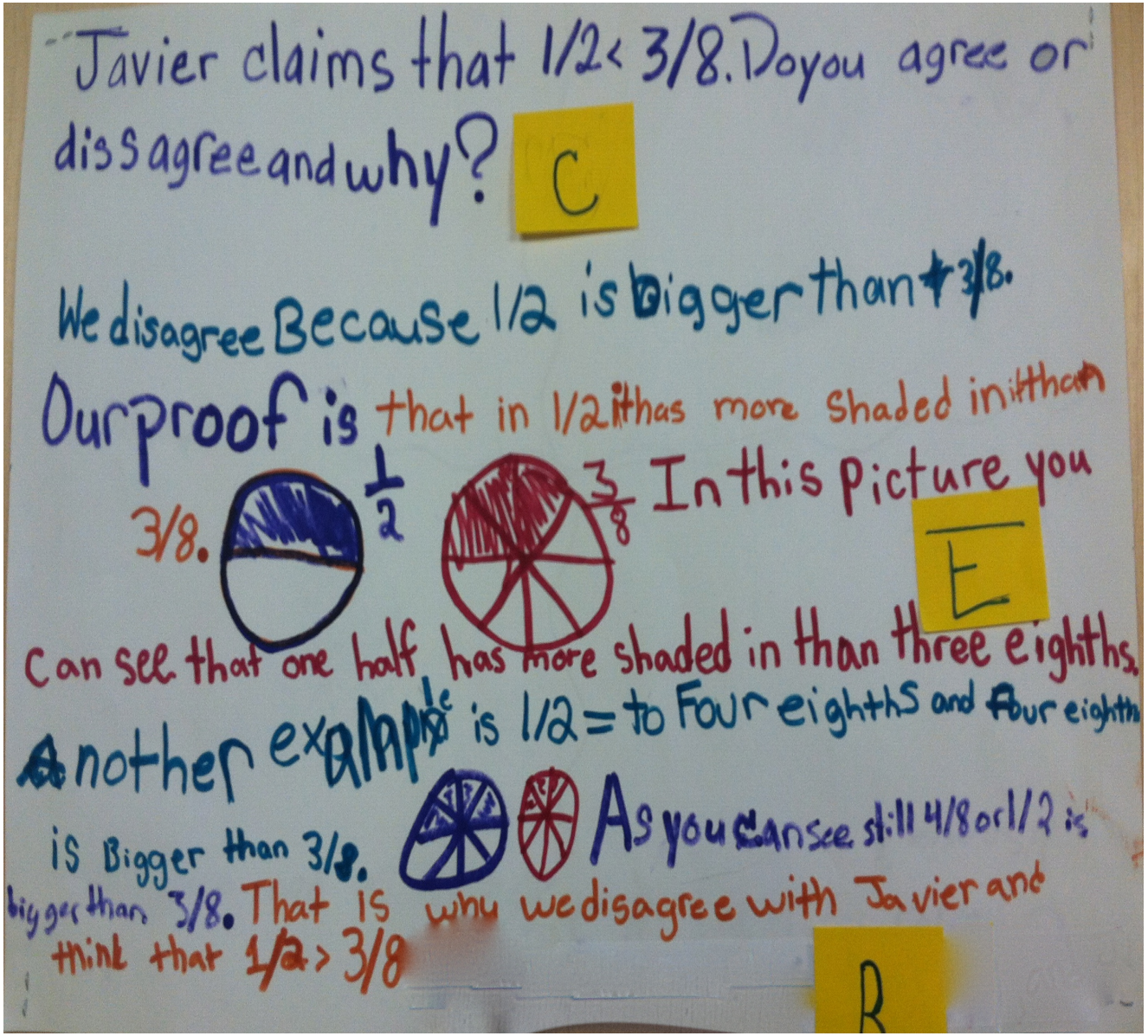
Student 2


Javier claims that 1/2 < 3/8. Do you agree or disagree?
C
We disagree with Javier because if we have a cookie and we ate half of it we would have half left
But if we ate 4/8 of the cookie it would just be an equivalent fraction to 1/2.
E
Is the strongest
Missing Parts
1/2
3/8
E
4/8
1/2
=
Equivalent fraction

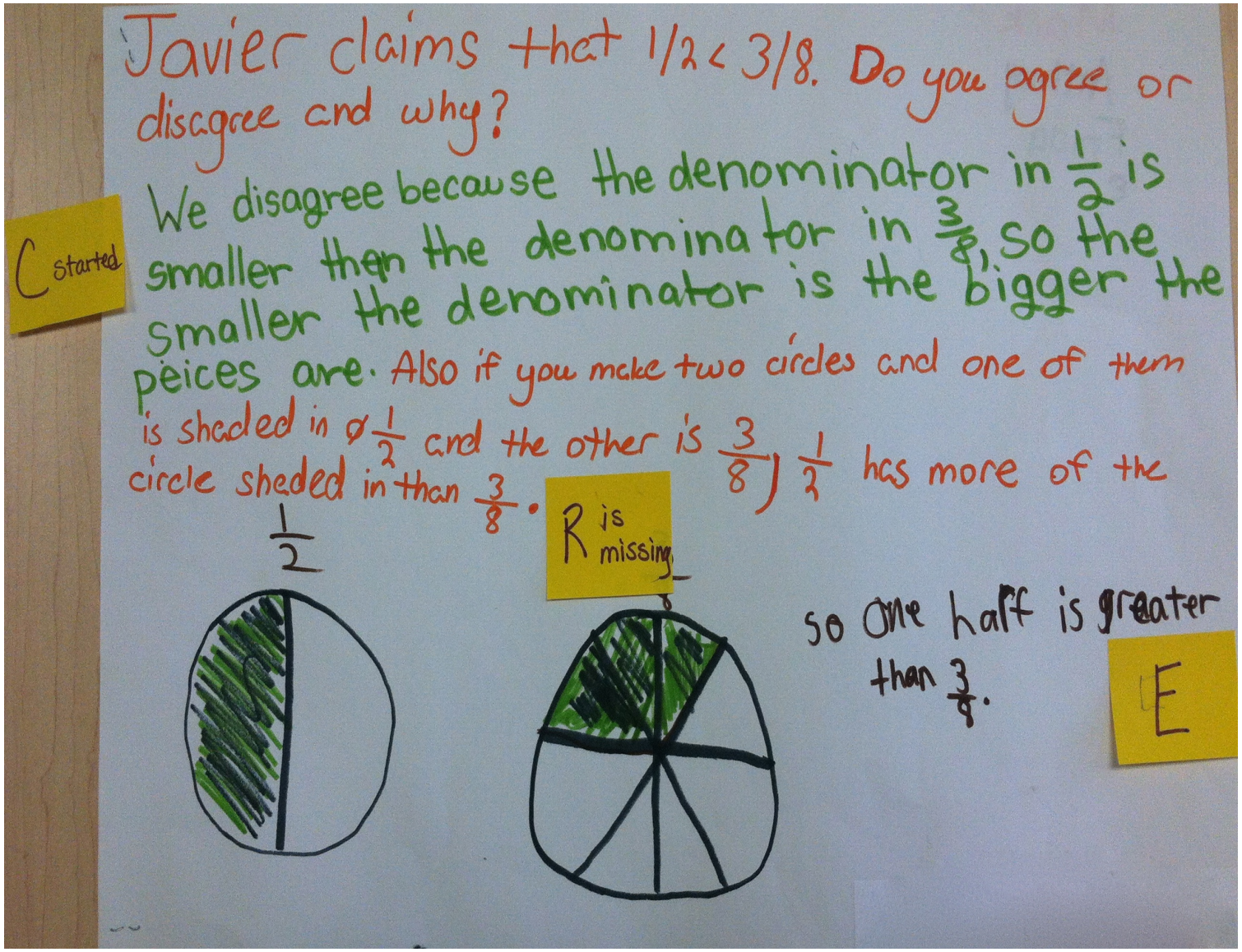
Student 3


Javier claims 1/2 < 3/8. Do you agree or disagree and why?
We disagree because if you look at our picture it shows 3/8 is less than 1/2. If you ADD ONE MORE 1/8 to 3/8 THEN IT WILL EQUAL 1/2.
If you added 1 more 1/8 to 4/8 Then it would be more than 1/2.
1/2 = 4/8
1/2
5/8
1/2
3/8
No Evidence
W/R

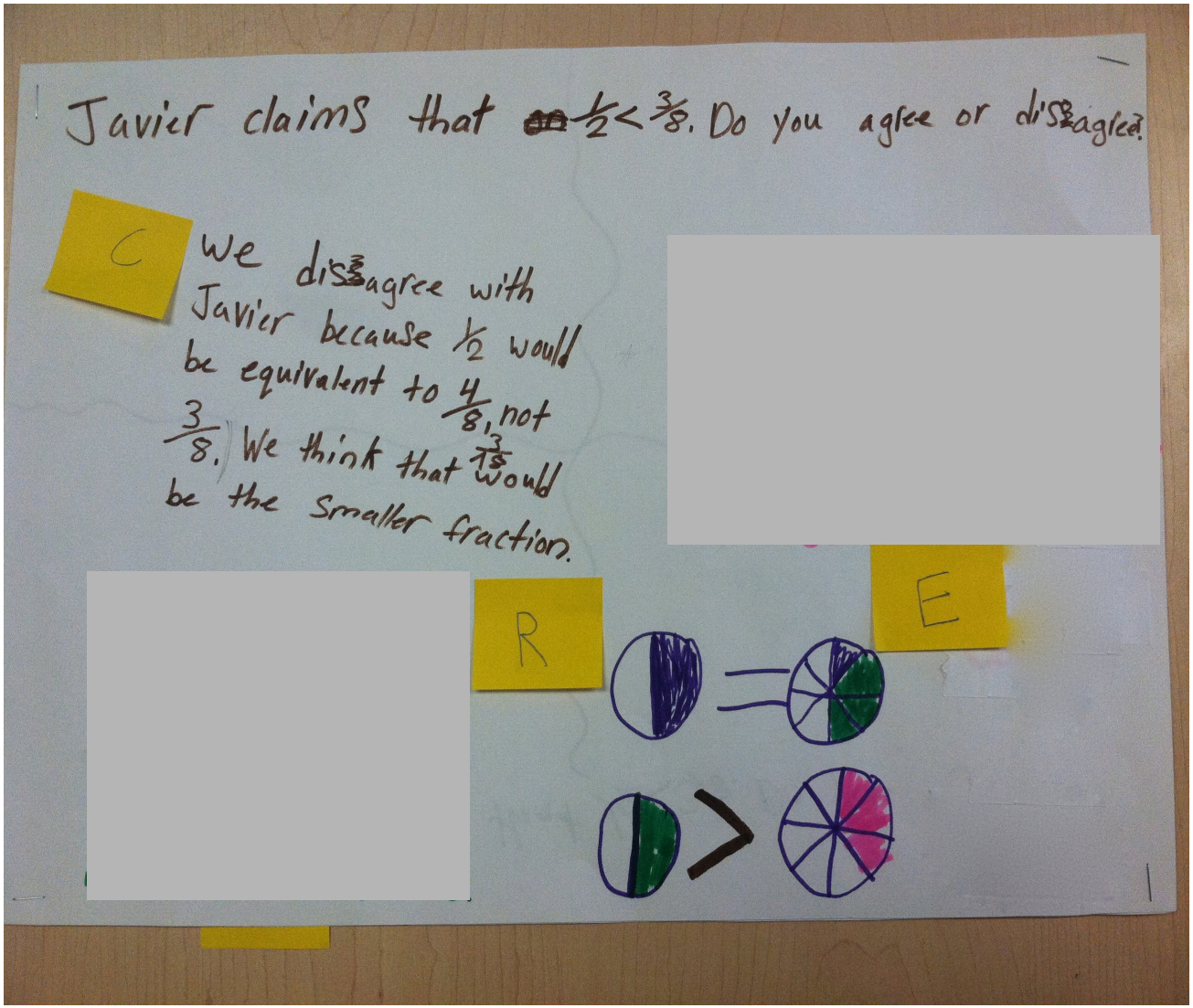
Student 4


Javier claims that 1/2 < 3/8. Do you agree or disagree and why?
C
We disagree Because 1/2 is bigger than 3/8.
Our proof is that in 1/2 it has more shaded in it than 3/8.
1/2
3/8
In this picture you can see that one half has more shaded in than three eighths.
E
Another example is 1/2 = to Four eighths and four eighths is Bigger than 3/8.
As you can see still 4/8 or 1/2 is bigger than 3/8. That is why we disagree with Javier and think that 1/2 > 3/8
R

Student 5


Javier claims that 1/2 < 3/8. Do you agree or disagree and why?
C started
We disagree because the denominator in 1/2 is smaller than the denominator in 3/8, so the smaller the denominator is the bigger the peices are. Also if you make two circles and one of them is shaded in of 1/2 and the other is 3/8 1/2 has more of the circle sheded in than 3/8.
1/2
R is missing
so one half is greater than 3/8.
E

Student 6


Javier claims that ½ < ⅜. Do you agree or disagree?
C
We disagree with Javier because ½ would be equivalent to 4/8, not 3/8. We think that ⅜ would be the smaller fraction.
R
E

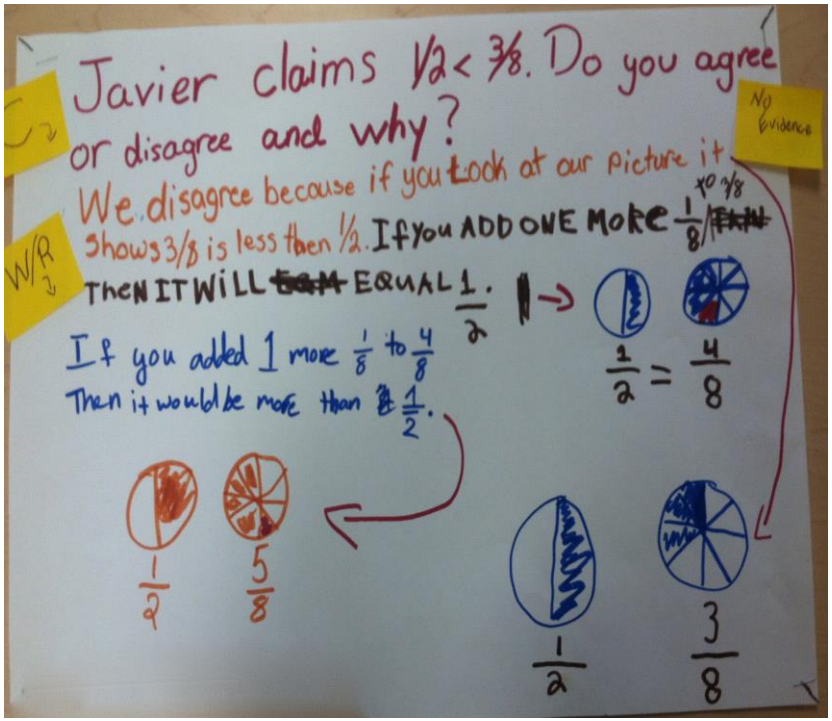
Work Samples Classification and Commentaries

Task: Comparing Fractions, Grade 5

Important note: The teachers and project members that discussed these work samples were not always unanimous in their determinations of quality. Although we might even agree on what the student did do, did not do, and strengths of the argument, there were differences in how much "weight" people put on different strengths and weaknesses. Thus, two teachers might see the same things in the student work sample, but one might want to classify the argument as, say, adequate quality and the other as low quality. This points to the importance of professional *discussions* and talking through the work samples with colleagues. There is no one absolute answer to whether a student work sample is high, adequate or low. Rather, trying to do the categorization leads to important conversations and helps a group clarify strengths, weaknesses, and what we value. That said, the teams reviewing these work samples had focused on argumentation for a year and had some level of shared vision for this work which we think is helpful to share and is reflected in the commentaries.

A Key linking the work samples from this ordered set with the sorting packet appears at the end of the document.

Student A



Commentary

*This argument is considered **High quality**.*

The students' claim is that they disagree with Javier. They use a pictorial representation to show that 3/8 is less than 1/2. They explain in words that if they add 1/8 to 3/8 it would equal 1/2.

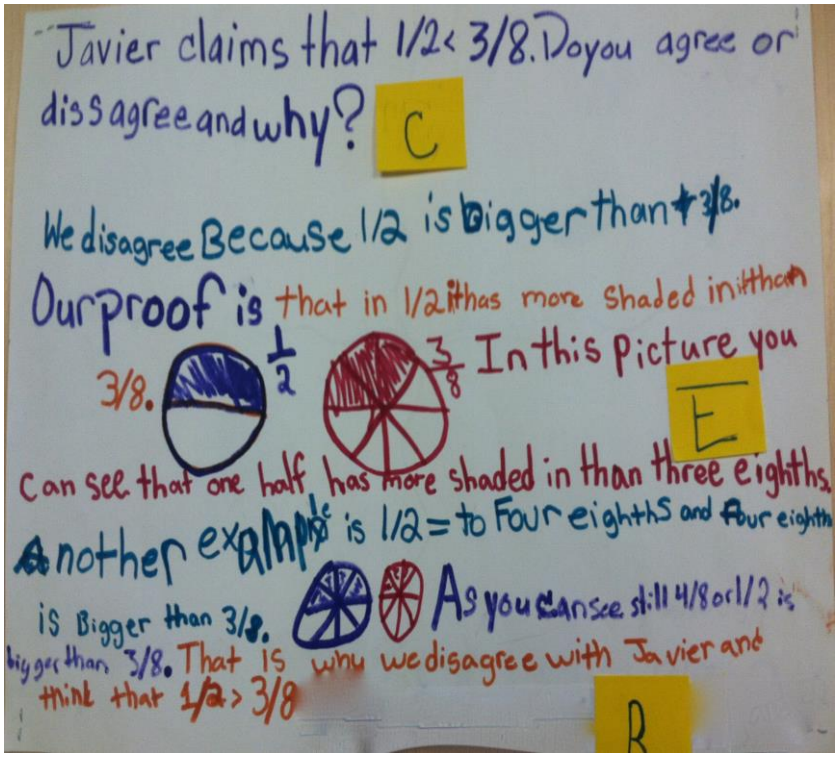
The response could be extended by including a statement explaining in words that 1/2 and 4/8 are equivalent fractions. The pictorial representation that compares 1/2 and 5/8 could be elaborated on to show the relationship with the comparison of 1/2 and 3/8.

Note: several students collaborated on the creation of this argument therefore it contains more than one way to support the claim but this is not necessary for a complete mathematical argument.

Argumentation Components

Claim	**Evidence**
The claim is clearly stated: "We disagree."	The students drew a pictorial representation of 1/2 and 3/8 and it clearly shows that 1/2 has more shaded. They also included a pictorial representation of 1/2 and 4/8 to show equivalence. They then state that 3/8 is 1/8 less than 1/2.
Warrants	**Language & Computation**
The students explicitly state that "If you add one more 1/8 to 3/8 then it will equal 1/2."	*All mathematical computations are correct and statements are true.*

Student B



Commentary

*This argument is categorized as **High quality**.*

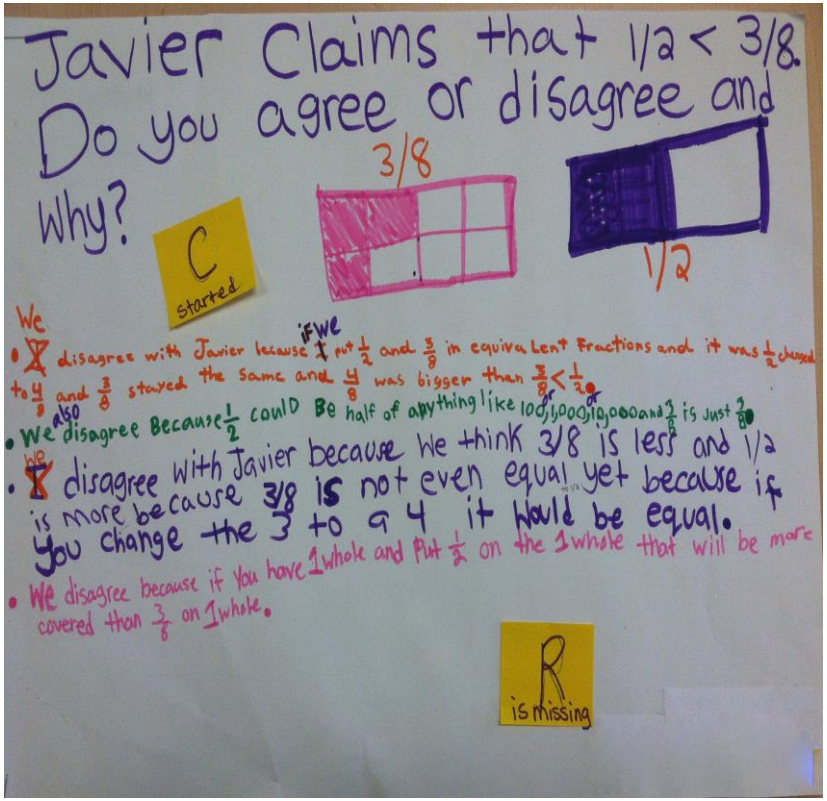
The students' claim is that they disagree with Javier. The students give two pieces of evidence and a warrant. The first piece of evidence is a pictorial representation of 1/2 compared to 3/8. The second piece of evidence is a picture that shows the comparison of ⅜ and 4/8, and a statement that 1/2 = 4/8 and 4/8 is bigger than 3/8. The warrant, "As you can see still 4/8 or 1/2 is bigger than 3/8," links back to the claim.

This response could be extended by including specific math vocabulary, and more precise language related to fractions (bigger = greater) Note: several students collaborated on this argument therefore there is repetitive information.

Argumentation Components

Claim	**Evidence**
The claim is stated "we disagree…"	*The fraction 1/2 is represented as an equivalent fraction with denominator of 8 as 4/8. The students state that 1/2 is greater than 3/8 as a result of comparing 4/8 and 3/8.* *Pictures are used to show the equivalence and the comparison.*
Warrants	**Language & Computation**
The students explicitly state that 1/2 is bigger than 3/8 because 4/8 (which is equivalent to 1/2) is bigger than 3/8.	*Explanations of diagrams are accurate and grade level appropriate.*

Student C



Commentary

*This argument is categorized as **Adequate quality**.*

The students' claim is that they disagree with Javier. They use a pictorial representation to show that 3/8 is less than ½. They also explain using words that 3/8 is less than ½ when compared to one whole.

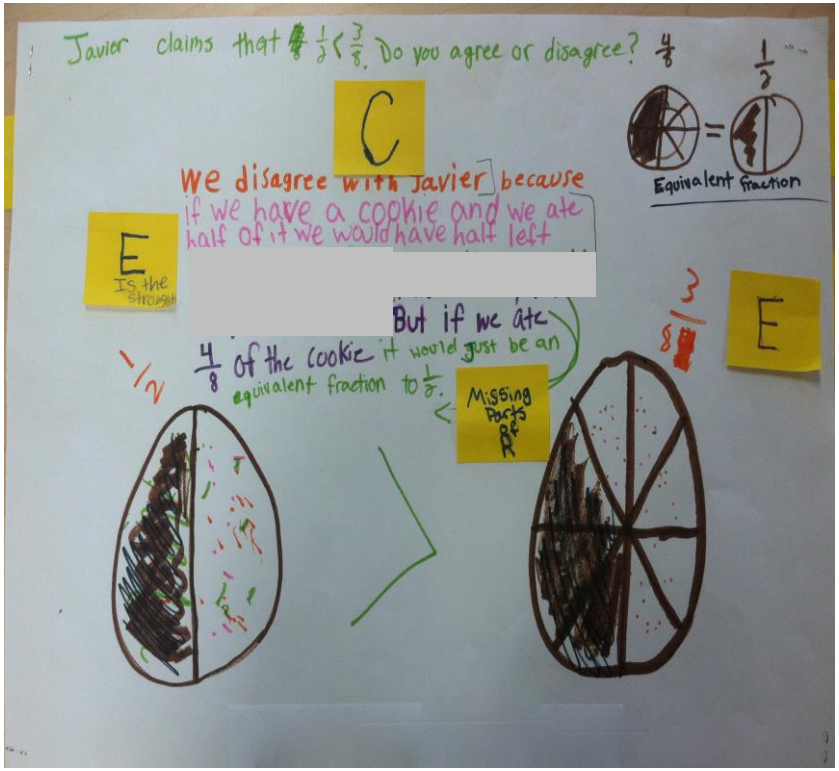
The argument could be strengthened by combining the ideas presented in all bullets but the second one. The pictorial representation communicates the ideas in a clear manner, but could be strengthened by showing eights in the picture representing 1/2.

It is unclear why the students have the second bullet point. It should be omitted as it does not connect well with the rest of the pieces in this argument and shows a misunderstanding.

Argumentation Components

Claim	**Evidence**
The claim is clearly stated: "We disagree with Javier."	*The students show a pictorial representation of 1/2 and 3/8, and the picture clearly shows that 1/2 has more shaded. They then state that if you consider 1/2 as 4/8, then it is clear that 3/8 is less than 1/2 or 4/8.*
Warrants	**Language & Computation**
The warrants are implicit in the use of equivalent fractions and relying on areas to compare the values of the fractions.	*With the exception of the second and third bullet points, all mathematical computations and statements are true. The third bullet needs to be polished and the second bullet shows a misconception.*

Student D



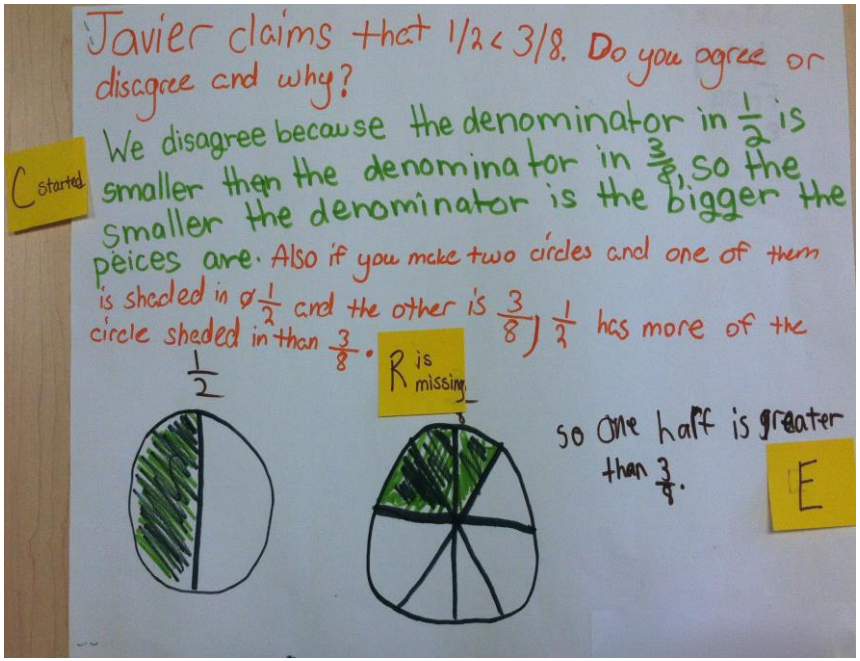
Commentary

*This argument is considered **Adequate quality**.*
The students claim that they disagree with Javier and use a model as evidence to communicate reasoning. The students show each fraction as part of a circle. The student shows understanding of the equivalence between 1/2 and 4/8 and use appropriate labels and vocabulary to represent each fraction (as part of a whole circle) correctly, as well as equivalence. However, the students need a stronger connection between the visual pictures and state an explicit warrant that does not rely on the cookie story. The visual implies that 1/2 is greater than 3/8, but the link between the two visuals is missing (that 3/8 is 1/8 less than 4/8). In regards to the warrant, the explanation does not link directly to the visual evidence, and is open for misinterpretation.

Argumentation Components

Claim	**Evidence**
The claim is stated "we disagree with Javier".	*Students use diagrams to compare the two fractions. Each fraction is accurately represented visually and is correctly labeled. Pictures are used to show that 1/2 is greater than 3/8 which support the claim.*
Warrants	**Language & Computation**
The implicit warrant is offered in the form of a story about cookies.	*The visual representations are labeled correctly using appropriate mathematical vocabulary such as equivalent fraction. However, the cookie explanation is unclear, and could be interpreted incorrectly.*

Student E



Commentary

*This argument is categorized as **Low quality**.*

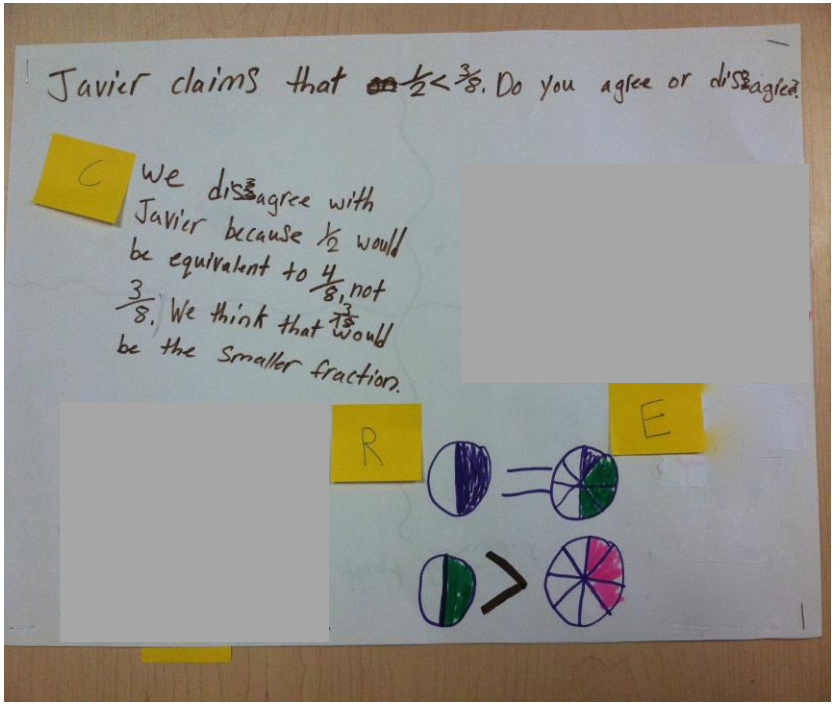
The students' claim is that they disagree with Javier. They correctly use a pictorial representation to show that 3/8 is less than 1/2. However, the argument needs to be strengthened by clarifying the first statement, where they focus solely on the denominators to compare the fractions. This focus on denominators may indicate a misunderstanding of how fractions should be interpreted.

The argument could be strengthen by better supporting the sole focus son denominators. For example, students could use the equivalency between 1/2 and 4/8 and relating that back to 3/8. In addition, more accurate mathematical vocabulary could make the reasoning clearer.

Argumentation Components

Claim	**Evidence**
The claim is stated: "We disagree."	*The students show a pictorial representation of 1/2 and 3/8. They explain that one half of the circle covers more area than 3/8.*
Warrants	**Language & Computation**
The warrant for the beginning sentence relies on the comparison of the denominators, which does not completely support the argument. The warrant related to the picture is missing.	*The first sentence is inaccurate. Language used to describe the pieces is vague: "bigger pieces". Similarly for the description of the shaded areas "more of the circle shaded".*

Student F



Commentary

*This argument is considered **Low quality**.*

*This students' claim is that they disagree with Javier, and support this claim with the statement that 1/2 would be equivalent to 4/8, not 3/8. There is no comparison made between 1/2 and 3/8 other than that they are not equivalent. There is not enough justification to support that 3/8 is the **smaller** fraction due to the relationship of 4/8 and 1/2.*

The students provide visual representations (without labels) that imply an understanding but this is not linked back to the claim.

The argument could be strengthened by noting how equivalent fractions and the comparison of 4/8 and 3/8 can be combined to support their statement that 3/8 is smaller, or by explaining how the visuals to support the statement.

Argumentation Components

Claim	**Evidence**
The claim is stated: "We disagree with Javier."	*Students offer two visual representations: one that shows ½ as equivalent to 4/8, the other that shows that ½ is bigger than 3/8. Students also state that 1/2 is equivalent to 4/8.*
Warrants	**Language & Computation**
Warrant is "1/2 would be equivalent to 4/8, not 3/8". However, this is not sufficient.	*Further explanation is necessary to make the reasoning clearer. Visuals are missing appropriate labels and sufficient support.*

Key Connecting Sorting Packet to Argumentation Resource Packet

Student number (Soring Packet)	Resource Packet Sample
1	C (Adequate)
2	D (Adequate)
3	A (High)
4	B (High)
5	E (Low)
6	F (Low)

Student number (Soring Packet)	Resource Packet Sample (category)
3	A (High)
4	B (High)
1	C (Adequate)
2	D (Adequate)
5	E (Low)
6	F (Low)